

Ultra Fast Self-Compton Cooling

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We investigate the synchrotron self-Compton process (in which seed photons for inverse Compton emission is supplied by synchrotron emission) in the plane-parallel shell taking the shock structure into account. We find that in fast cooling (when the electron cooling time Δt_{cool} is shorter than the shock crossing time Δt_{dyn}) the one-zone approximation underestimates the energy density of the seed soft photons by a factor of $\sim \ln(\Delta t_{dyn}/\gamma \Delta t_{cool})$, where γ is the Lorentz factor of the shock in the shocked fluid frame. This factor may be order of unity (e.g., $\ln f \sim \ln 10^6 \sim 14$ in gamma-ray bursts) so that inverse Compton may cool electrons “ultra” fast in such as gamma-ray bursts, blazars and microquasars.

I. INTRODUCTION

Relativistic shocks often arise in astrophysics when a faster flow hits upon a slower one such as in gamma-ray bursts (GRBs) (e.g., Piran 1999), blazars (e.g., Inoue & Takahara 1996; Kino, Takahara & Kusunose 2002), microquasars (e.g., Levinson & Waxman 2001; Kaiser, Sunyaev & Spruit 2000) and so on. In the relativistic shocks, the kinetic energy of the flow turns into the internal one, and some fraction of the internal energy is distributed to electrons and magnetic fields. The electrons are accelerated in the shock front to a power law distribution, while the magnetic fields are amplified by the shocks. Under these conditions, the accelerated electrons radiate nonthermal emission, such as synchrotron and inverse Compton emission. In this Letter we will consider the synchrotron self-Compton process, in which synchrotron photons from the electrons are the seed photons to be scattered.

The ratio of the inverse Compton to synchrotron luminosity Y is one of the most important quantity. Since the nonthermal spectrum of the GRBs or the blazars indicates that the sources are optically thin, only a negligible fraction of synchrotron photons is scattered. However, depending on the parameters, the Compton to synchrotron ratio Y can be larger than unity so that the ratio Y is essential for the estimate of the total energy. In fact, for example, if the TeV signal detected by the Milagro group is truly from GRB 970417a, the TeV fluence must be at least 10 times greater than the sub-MeV fluence of this GRB (Atkins et al. 2000). The ratio Y also affect the cooling time of the electrons. Furthermore the ratio Y may be important for the plasma physics, if we inversely estimate the physical parameters from the observed ratio of the inverse Compton to synchrotron luminosity.

So far the Compton to synchrotron ratio Y has been estimated from the one-zone argument as follows (Sari, Narayan & Piran 1996; Panaitescu & Kumar 2000; Sari & Esin 2001). Let us consider hereafter the fast cooling regime, that is, all the electron energy is radiated away within a dynamical time, and only single scatterings assuming that the higher order inverse Compton is suppressed by the Klein-Nishina effect (Rybicki & Lightman

1979). Assuming the isotropic distribution of electrons, the ratio Y is equal to the ratio of the energy density in soft radiation to magnetic field energy density (Rybicki & Lightman 1979). Then, we have

$$Y \equiv \frac{L_{IC}}{L_{syn}} = \frac{U_\gamma}{U_B} = \frac{U_{syn}}{U_B} = \frac{U_e/(1+Y)}{U_B} = \frac{\epsilon_e}{\epsilon_B(1+Y)}, \quad (1)$$

where U_γ , U_{syn} , U_B and U_e are the energy density of the seed photons, the synchrotron radiation, the magnetic fields, and the relativistic electrons, respectively. The parameters ϵ_e and ϵ_B are fractions of shock energy that go into the electrons and the magnetic energy, respectively. The solution of the equation (1) is

$$Y = \frac{-1 + \sqrt{1 + 4\epsilon_e/\epsilon_B}}{2}, \quad (2)$$

so that we have

$$Y = \begin{cases} \frac{\epsilon_e}{\epsilon_B}, & \text{if } \frac{\epsilon_e}{\epsilon_B} \ll 1, \\ \left(\frac{\epsilon_e}{\epsilon_B}\right)^{1/2}, & \text{if } \left(\frac{\epsilon_e}{\epsilon_B}\right)^{1/2} \gg 1. \end{cases} \quad (3)$$

At present the parameters ϵ_e and ϵ_B are highly uncertain.

In this Letter, we will show that the formula (3) is not applicable when the shock crossing time Δt_{dyn} is much longer than the electron cooling time Δt_{cool} , i.e., in the fast cooling case $f \equiv \Delta t_{dyn}/\Delta t_{cool} \gg 1$. In this case we will find that the formula is given by

$$Y = \begin{cases} \frac{\epsilon_e \ln(f/\gamma)}{\epsilon_B}, & \text{if } \frac{\epsilon_e \ln(f/\gamma)}{\epsilon_B} \ll 1, \\ \left[\frac{\epsilon_e \ln(f/\gamma)}{\epsilon_B}\right]^{1/2}, & \text{if } \left[\frac{\epsilon_e \ln(f/\gamma)}{\epsilon_B}\right]^{1/2} \gg 1, \end{cases} \quad (4)$$

taking the shock structure into account. Here γ is the Lorentz factor of the shock in the shocked fluid frame.

The correction factor $\ln(f/\gamma)$ in equation (4) may be order of unity. For example, let us consider the internal shocks of the GRBs (e.g., Piran 1999). The photon energy density may be estimated as $U_\gamma \sim L/4\pi r^2 c \Gamma^2 \sim 10^{10} L_{52} r_{13}^{-2} \Gamma_2^{-2} \text{ erg cm}^{-3}$ where $L = 10^{52} L_{52} \text{ erg s}^{-1}$ is the observed luminosity, $r = 10^{13} r_{13} \text{ cm}$ is the distance from the center at which the internal shocks take place, and $\Gamma = 10^2 \Gamma_2$ is the Lorentz factor of the shell. Then the cooling time is given by $\Delta t_{cool} \sim 3\gamma_e m_e c^2 / (4\sigma_T c \gamma_e^2 U_B) \sim$

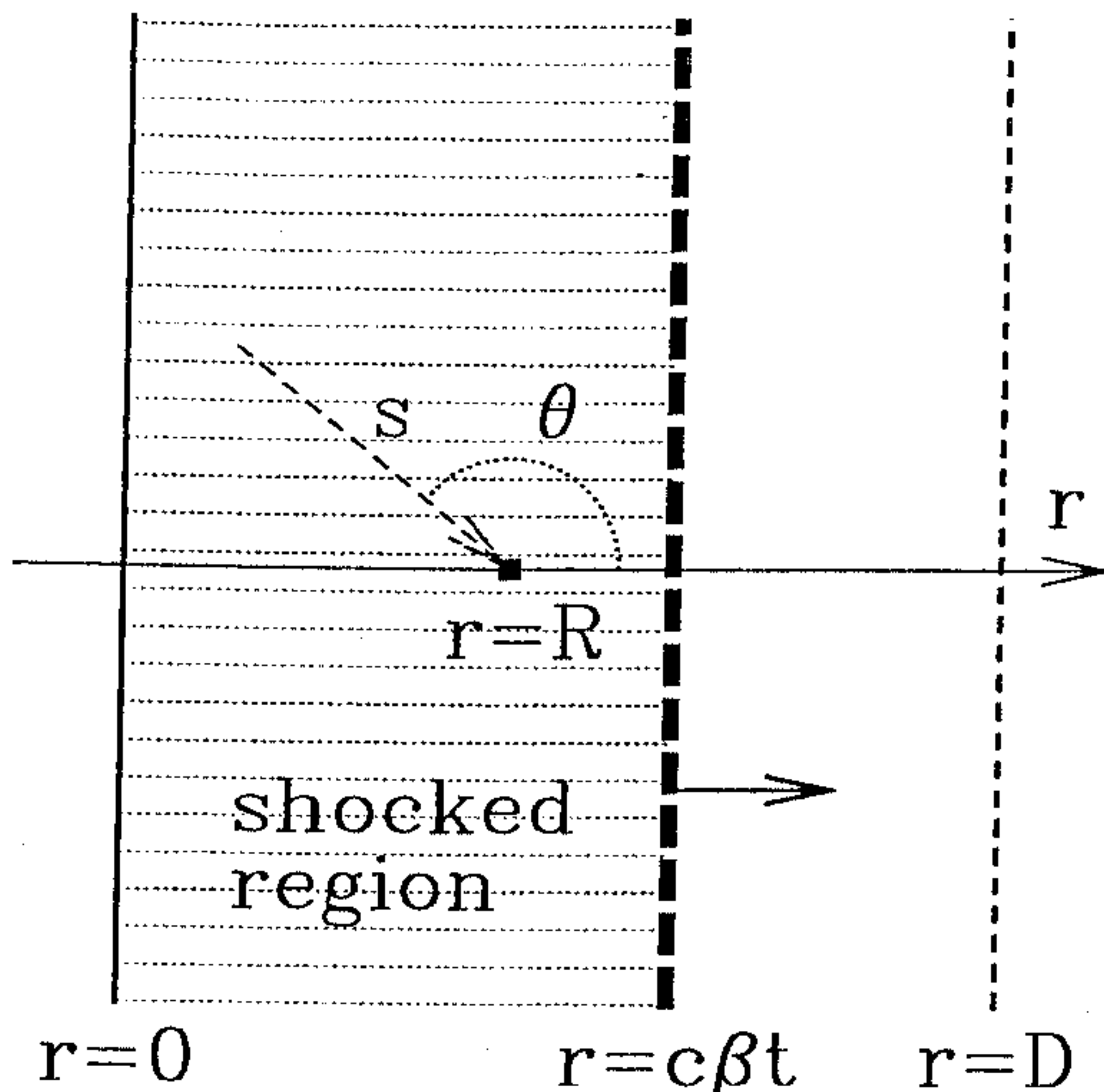


FIG. 1: Our simple model is shown. A uniform shell has a thickness D and is optically thin. A shock is propagating with a velocity $c\beta = c(1 - \gamma^{-2})^{1/2}$ measured in the shocked fluid frame. Electrons are accelerated right behind the shock, and after that they cool radiating synchrotron and Compton emission. The motion of the electrons is neglected in the shocked fluid frame.

$10^{-6} Y \gamma_{e,3}^{-1} L_{52}^{-1} r_{13}^2 \Gamma_2^2$ s, where $\gamma_e = 10^3 \gamma_{e,3}$ is the Lorentz factor of the electrons. On the other hand, the shell crossing time in the fluid frame may be given by $\Delta t_{dyn} \sim \Gamma d/c \sim 1 \Gamma_2 d_8$ s, where $d = 10^8 d_8$ cm is the shell thickness in the lab frame. Thus the correction factor is about $\ln(f/\gamma) \sim \ln(\Delta t_{dyn}/\Delta t_{cool}) \sim \ln 10^6 \sim 14$, where we assume $\gamma \sim 1$. Note that the electron acceleration time $\Delta t_{acc} \sim \gamma_e m_e c^2 / ceB \sim 10^{-8} \gamma_{e,3} Y^{1/2} L_{52}^{-1/2} r_{13} \Gamma_2$ s is much shorter than two other timescales Δt_{dyn} and Δt_{cool} .

II. SIMPLE MODEL

Let us consider the following simple model. The extension to more complicated models is straightforward. We consider an uniform shell with a thickness D and a shock propagating with a velocity $c\beta = c(1 - \gamma^{-2})^{1/2}$ measured in the shocked fluid frame (see Figure 1). We assume that electrons are accelerated just behind the shock and that the motion of the electrons is negligible in the shocked fluid frame, since the electron acceleration time Δt_{acc} is typically much shorter than other timescales.

The energy density of soft photons at a point $r = R$ is given by the integration of all synchrotron radiation from the accelerated electrons as

$$U_\gamma(t, r = R) = \frac{2\pi}{c} \int d\mu ds \frac{1}{4\pi} P_{syn} \left(t - \frac{s}{c}, r \right), \quad (5)$$

where $P_{syn}(t, r)$ erg s⁻¹cm⁻³ is the synchrotron power

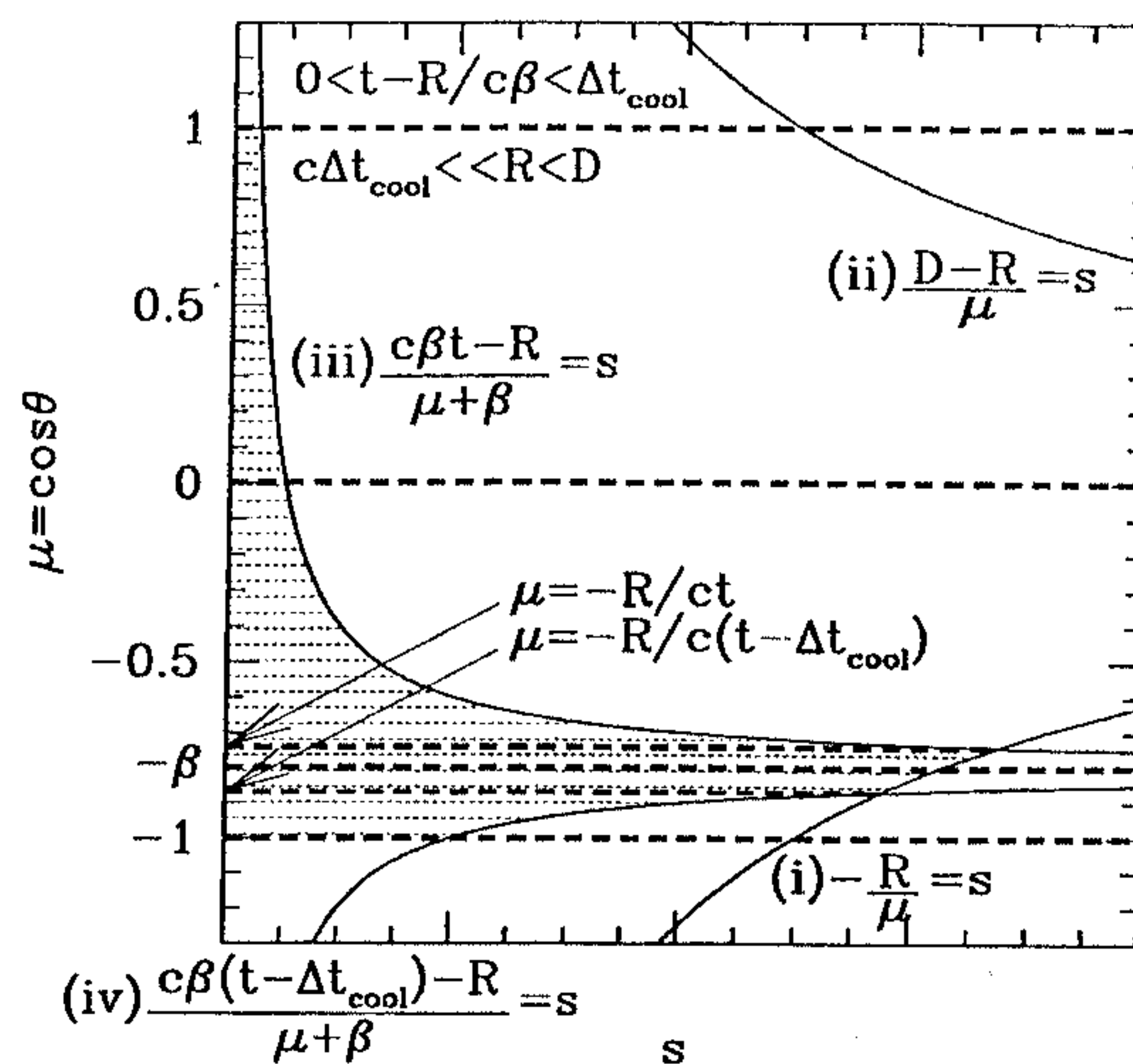


FIG. 2: The region to be integrated in equation (5) is shaded in the (s, μ) plane for $0 < \tilde{t} \equiv t - R/c\beta < \Delta t_{cool}$ and $c\Delta t_{cool} \ll R \lesssim D$. Four constraints for the integral are shown by solid lines, (i) $-R/\mu = s$, (ii) $(D - R)/\mu = s$, (iii) $(c\beta t - R)/(\mu + \beta) = s$ and (iv) $[c\beta(t - \Delta t_{cool}) - R]/(\mu + \beta) = s$. The lines (i) and (iii) cross at $\mu = -R/ct$, and the lines (i) and (iv) cross at $\mu = -R/c(t - \Delta t_{cool})$.

at a time t and a position r , and $\mu = \cos \theta$. Note that the retarded time $t - s/c$ is used in equation (5). To be precise, we have to solve the electron cooling together with equation (5). However, to extract the essence, we adopt here the following simple functional form as the synchrotron power,

$$P_{syn}(t, r) = A_{syn} \left[H \left(t - \frac{r}{c\beta} \right) - H \left(t - \frac{r}{c\beta} - \Delta t_{cool} \right) \right], \quad (6)$$

which means that electrons radiate with a constant power A_{syn} erg s⁻¹ cm⁻³ for a duration Δt_{cool} s. This is sufficient for the following argument.

To compare the energy density $U_\gamma(t, r)$ with the one-zone estimate U_γ^{1zone} , we have to take the time and position averaging $\langle U_\gamma(t, r) \rangle$. The one-zone approximation of equation (1) is equivalent to setting $\langle U_\gamma(t, r) \rangle = \int dt dr P_{syn}(t, r)/D$, which is not satisfied in general. With equation (6), the photon energy density of one-zone approximation is given by

$$U_\gamma^{1zone} = U_{syn} = \int dt P_{syn}(t, r) = A_{syn} \Delta t_{cool}. \quad (7)$$

Equation (6) makes it possible to integrate equation (5) analytically. In Figure 2, the region to be integrated is shaded. There are four constraints for the integral in equation (5), which are shown by four solid lines in Figure 2. (i) $-R/\mu > s$ so that the shell has an end $r = R + s\mu > 0$. (ii) $(D - R)/\mu > s$ so that the shell has

an end $r = R + s\mu < D$. (iii) $(c\beta t - R)/(\mu + \beta) > s$ for the first Heaviside step function in equation (6) to be unity at the retarded time $t - s/c$, i.e., $t - s/c - (R + s\mu)/c\beta > 0$. (iv) $[c\beta(t - \Delta t_{cool}) - R]/(\mu + \beta) > s$ for the second step function in equation (6) to be zero at the retarded time $t - s/c$, i.e., $t - \Delta t_{cool} - s/c - (R + s\mu)/c\beta < 0$. When electrons radiate, $0 < \tilde{t} \equiv t - R/c\beta < \Delta t_{cool}$, fast, $c\Delta t_{cool} \ll R \lesssim D$ (more precisely $c\beta\Delta t_{cool} \ll (1 - \beta)R$ and $c\beta\Delta t_{cool} \ll (1 + \beta)(D - R)$), the shaded region in Figure 2 can be integrated as

$$\begin{aligned}
 U_\gamma(t, R) &= \frac{A_{syn}}{2c} \left[\int_{-1}^{-R/c(t - \Delta t_{cool})} d\mu \frac{c\beta(t - \Delta t_{cool}) - R}{\mu + \beta} \right. \\
 &+ \left. \int_{-R/c(t - \Delta t_{cool})}^{-R/ct} d\mu \left(-\frac{R}{\mu} \right) + \int_{-R/ct}^1 d\mu \frac{c\beta t - R}{\mu + \beta} \right] \\
 &= \frac{A_{syn}}{2} \left[\beta(\Delta t_{cool} - \tilde{t}) \ln \frac{(1 - \beta)[R + c\beta(\tilde{t} - \Delta t_{cool})]}{c\beta^2(\Delta t_{cool} - \tilde{t})} \right. \\
 &+ \left. \frac{R}{c} \ln \frac{R + c\beta\tilde{t}}{R + c\beta(\tilde{t} - \Delta t_{cool})} + \beta\tilde{t} \ln \frac{(1 + \beta)(R + c\beta\tilde{t})}{c\beta^2\tilde{t}} \right] \\
 &\simeq \frac{\beta A_{syn}}{2} \left[(\Delta t_{cool} - \tilde{t}) \ln \frac{(1 - \beta)R}{c\beta^2(\Delta t_{cool} - \tilde{t})} \right. \\
 &+ \left. \Delta t_{cool} + \tilde{t} \ln \frac{(1 + \beta)R}{c\beta^2\tilde{t}} \right], \quad (8)
 \end{aligned}$$

where we use $c\Delta t_{cool} \ll R$ in the last equality. After taking the time and position average, $\langle U_\gamma(t, r) \rangle = \int_0^{\Delta t_{cool}} d\tilde{t} \int_0^D dr U_\gamma(t, r)/\Delta t_{cool} D$, we have

$$\langle U_\gamma(t, r) \rangle = \frac{\beta A_{syn} \Delta t_{cool}}{2} \left[\frac{1}{2} + \ln \frac{f}{\beta\gamma} \right], \quad (9)$$

where $f \equiv D/c\beta\Delta t_{cool}$ is the ratio of the shock crossing time to the electron cooling time. Therefore, in the fast cooling case, $\ln(f/\gamma) \gg 1$, the one-zone approximation in equation (7) underestimates the ambient soft photons by a factor of $\sim \ln(f/\gamma)$, i.e.,

$$\langle U_\gamma(t, r) \rangle \sim U_\gamma^{1zone} \times \ln(f/\gamma), \quad (10)$$

for $\beta \sim 1$. The logarithmic term originates from the integral of the term $1/\mu$. This is similar to the well known fact that the infinite planar problem of the radiative transfer has an logarithmic divergence.

III. ULTRA FAST COOLING

Increasing the ambient soft photons enhances the inverse Compton emission. From equation (10), in the fast cooling case $\ln(f/\gamma) \gg 1$, the ambient photon energy density is more than that from the one-zone argument by a factor of $\sim \ln(f/\gamma)$, so that equation (1) should be

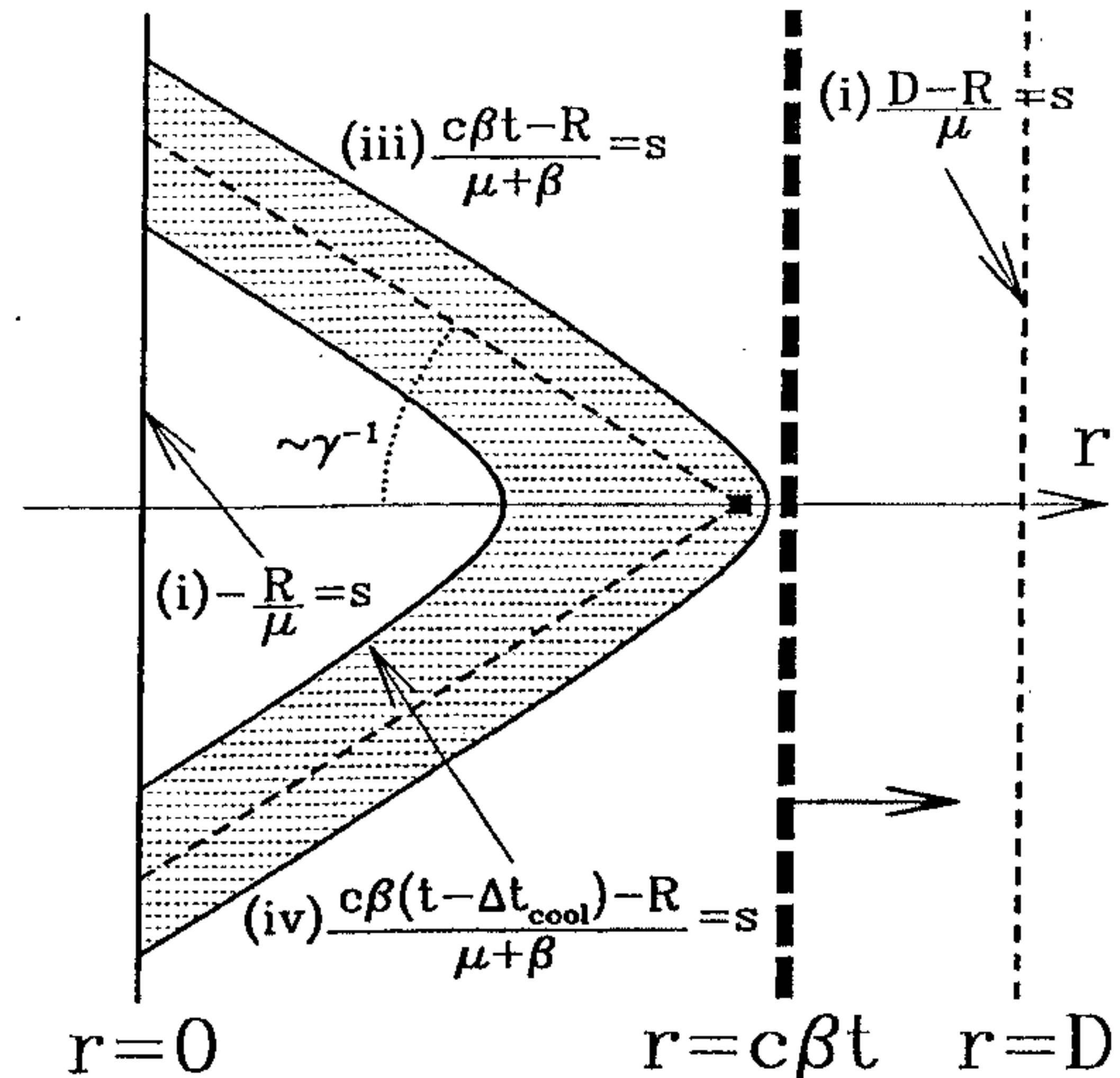


FIG. 3: The region from which the photons come to the square point is shade (see also Figure 1). This causal region is bounded by four constraints as in Figure 2, (i) $-R/\mu = s$, (ii) $(D - R)/\mu = s$, (iii) $(c\beta t - R)/(\mu + \beta) = s$, and (iv) $[c\beta(t - \Delta t_{cool}) - R]/(\mu + \beta) = s$. The boundaries (iii) and (iv) represent the hyperboloid of two sheets. The boundary (iv) is not relevant to this figure. Approximately, photons come from the direction $\tan(\pi - \theta) = \beta^{-1}\gamma^{-1}$, which is shown by the dashed line.

modified as

$$Y = \frac{U_\gamma}{U_B} = \frac{U_{syn} \ln(f/\gamma)}{U_B} = \frac{\epsilon_e \ln(f/\gamma)}{\epsilon_B(1 + Y)}. \quad (11)$$

The solution of the above equation is

$$Y = \frac{-1 + \sqrt{1 + 4\epsilon_e \ln(f/\gamma)/\epsilon_B}}{2}, \quad (12)$$

so that we have equation (4).

To make the physical situation clear, the region from which the photons come is shade in Figure 3. This causal region is bounded by four constraints as in Figure 2, (i) $-R/\mu = s$, (ii) $(D - R)/\mu = s$, (iii) $(c\beta t - R)/(\mu + \beta) = s$ and (iv) $[c\beta(t - \Delta t_{cool}) - R]/(\mu + \beta) = s$. The boundary (iii) represents the hyperboloid of two sheets,

$$\frac{x^2 + y^2}{\gamma^2 c^2 \beta^2 \tilde{t}^2} - \frac{(r - R - \gamma^2 c\beta \tilde{t})^2}{\gamma^4 c^2 \beta^4 \tilde{t}^2} = -1, \quad (13)$$

where $x^2 + y^2 = s^2(1 - \mu^2)$. The boundary (iv) is also described by equation (13) with replacing \tilde{t} by $\tilde{t} - \Delta t_{cool}$. From equation (13) we can find that the photons approximately comes from the direction $\tan(\pi - \theta) \sim \pi - \theta \sim \beta^{-1}\gamma^{-1}$. This is physically reasonable since the shock front has the same velocity as the photons traveling in the direction $\tan(\pi - \theta) = \beta^{-1}\gamma^{-1}$.

The approximation of the infinite width of the shell is broken when the extent of the shell is smaller than $\sim \beta^{-1}\gamma^{-1}D$. In the internal shocks of the GRBs, for example, this plane approximation is good since the distance from the center at which the internal shocks take place $r = 10^{13}r_{13}$ cm is much larger than $\sim \beta^{-1}\gamma^{-1}D \sim \Gamma d \sim 10^{10}\Gamma_2 d_8$ cm.

IV. SUMMARY AND DISCUSSIONS

We have investigated the synchrotron self-Compton process taking the one dimensional shock structure into account. We have found that in the fast cooling case one-zone approximation underestimates the energy density of the seed photons by a factor of $\sim \ln(f/\gamma)$, where $f = \Delta t_{dyn}/\Delta t_{cool}$ is the ratio of the shock crossing time to the electron cooling time and γ is the Lorentz factor of the shock in the shocked fluid frame. This enhances the inverse Compton cooling (fast cooling becomes “ultra” fast cooling) so that the Compton parameter Y in equation (3) is larger than that of one-zone estimate in equation (4).

We have discussed the ultra fast cooling using the internal shocks of the GRBs as an example. This mechanism may be also important for (early) afterglows of the GRBs (e.g., Sari, Narayan & Piran 1996; Waxman

1997; Wei & Lu 1998; Panaitescu & Kumar 2000; Sari & Esin 2001; Wang, Dai & Lu 2001), blazars (Inoue & Takahara 1996; Kino, Takahara & Kusunose 2002) and microquasars (Levinson & Waxman 2001; Kaiser, Sunyaev & Spruit 2000) since the fast cooling stage likely exists. Our model may be too simple to describe these objects. For example, the plane-parallel approximation may not be good, we may have to take both forward and reverse shocks into account and so on. If the synchrotron power is not constant but a function of the distance from the center, the logarithmic correction factor may become a power law one. These are interesting future problems.

Also we have not dealt with the details of the spectrum. Ultra fast cooling may modifies the electron distribution conventionally used in one-zone approximation. We will be able to answer this question by solving one dimensional radiative transfer. This is also an interesting future problem.

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