

Gravitational Wave Astronomy with Short-Arm Space Interferometer

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Abstract.—

It may be possible to construct a laser interferometer gravitational wave antenna in space with $h_{rms} \sim 10^{-27}$ at $f \sim 0.1\text{Hz}$ in this century. We show possible specification of this antenna which we call DECIGO. Using this antenna we show that 1) typically 10^5 ($10^4 \sim 10^6$) chirp signals of coalescing binary neutron stars per year may be detected with $S/N \sim 10^4$. 2) We can directly measure the acceleration of the universe by ten years observation of binary neutron stars. 3) The stochastic gravitational waves of $\Omega_{GW} \gtrsim 10^{-20}$ predicted by the inflation may be detected by correlation analysis for which effects of the recent cosmic acceleration would become highly important. Our formula for phase shift due to accelerating motion might be also applied for binary sources of LISA. This proceeding is based on Ref. [1].

Specification of DECIGO.—

The sensitivity of a space antenna with an arm length of 1/10 of LISA [2] and yet the same assumption of the technology level, such as a laser power of 1 W, the optics of 30 cm, etc. will be $4 \times 10^{-21}\text{Hz}^{-1/2}$ around 0.1 Hz in terms of strain, a factor of 10 better than the planned LISA sensitivity around 0.1 Hz (see also <http://www.physics.montana.edu/maggie> for a project named MAGGIE around this band). The sensitivity could be improved by a factor of 1000 for the next generation of a space antenna with more sophisticated technologies such as implementation of higher-power lasers and larger optics in order to increase the effective laser power available on the detectors, and thus to reduce the shot noise. The ultimate sensitivity of a space antenna in the far future could be, however, 3×10^{-27} around 0.1 Hz in terms of strain, assuming the quantum limit sensitivity for a 100 kg mass and an arm length of 1/10 of LISA. We name this detector DECIGO. This requires an enormous amount of effective laser power, and also requires that the other noise sources, such as gravity gradient noise, thermal noise, practical noise, etc. should be all suppressed below the quantum noise. Here we assume that such an antenna may be available by the end of this century, although we note that within the next five years or so NASA will begin serious discussions of a follow-on to the planned NASA/ESA LISA mission, so DECIGO technology may be achieved sooner. Note here that when the pioneering efforts to detect the gravitational waves started in the last century using resonant type detectors as well as laser interferometers, few people expected the present achievement in resonant type

detectors such as IGEC(bar) [3] and in laser interferometers such as TAMA300 [4], LIGO, GEO600, and VIRGO (for these detectors see [5]). Therefore all the experimentalists and the theorists on gravitational waves should not be restricted to the present levels of the detectors. Our point of view in this Letter is believing the proverb “Necessity is the mother of the invention” so that we argue why a detector like DECIGO is necessary to measure some important parameters in cosmology.

The sensitivity of DECIGO, which is optimized at 0.1 Hz, is assumed to be limited only by radiation pressure noise below 0.1 Hz and shot noise above 0.1 Hz. The contributions of the two noise sources are equal to each other at 0.1Hz, giving the quantum limit sensitivity at this frequency. The radiation pressure noise has a frequency dependence of $\propto f^{-2}$ (in units of $\text{Hz}^{-1/2}$) because of the inertia of the mass, while the shot noise has a dependence of approximately $\propto f^1$ (in units of $\text{Hz}^{-1/2}$) because of the signal canceling effect due to the long arm length. In figure 1 we show sensitivity of various detectors and characteristic amplitude h_c for a chirping NS-NS binary at $z = 1$.

Direct Measurement of the Acceleration of the Universe.—

Recent distance measurements for high-redshift supernovae suggest that the expansion of our universe is accelerating [6] which means that the equation of the state of the universe is dominated by “dark energy” with $\rho + 3p < 0$. *SuperNova / Acceleration Probe* (SNAP, <http://lbl.gov>) project will observe ~ 2000 Type Ia supernovae per year up to the redshift $z \sim 1.7$ so that we may get the accurate luminosity distance $d_L(z)$ in near future. Gravitational wave would be also a powerful tool to determine $d_L(z)$ [7].

From accurate $d_L(z)$ one may think that it is possible to determine the energy density $\rho(z)$ and the pressure $p(z)$ as functions of the redshift. However as shown by Weinberg [8] and Nakamura & Chiba [9], $\rho(z)$ and $p(z)$ can not be determined uniquely from $d_L(z)$ but they depend on one free parameter Ω_{k0} (the spatial curvature).

Recent measurement of the first peak of the anisotropy of CMB is consistent with a flat universe ($\Omega_{k0} = 0$) for primordially scale-invariant spectrum predicted by slow-roll inflation [10] under the assumption of Λ cosmology. However it is important to determine the curvature of the universe irrespective of the theoretical assumption on the equation of the state and the primordial spectra also. In other words an independent determination of

Ω_{k0} is indispensable since Ω_{k0} is by far the important parameter. As discussed in [9], the direct measurement of the cosmic acceleration [11] can be used for this purpose. Here we point out that the gravitational waves from the coalescing binary neutron stars at $z \sim 1$ observed by DECIGO may be used to determine Ω_{k0} . Even in the worst case the redundancy is important to confirm such an important finding as the dark energy.

Cosmic Acceleration.—

A Cosmic Acceleration.— We consider the propagation of gravitational wave in our isotropic and homogeneous universe. The metric is given by $ds^2 = -dt^2 + a(t)^2(dx^2 + r(x)^2(d\theta^2 + \sin^2\theta d\phi^2))$, where $a(t)$ is the scale factor and $a(t)r(x)$ represents the angular distance. The relation between the observed time of the gravitational waves t_o at $x = 0$ and the emitted time t_e at the fixed comoving coordinate x is given by $\int_{t_e}^{t_o} \frac{dt}{a(t)} = x = \text{const.}$ Then we have $dt_o/dt_e = a_o/a_e = (1+z)$ and

$$\begin{aligned} \frac{d^2 t_o}{dt_e^2} &= (1+z)a_e^{-1}(\partial_t a(t_o) - \partial_t a(t_e)) \equiv g_{cos}(z) \\ &= (1+z)((1+z)H_0 - H(z)), \end{aligned} \quad (1)$$

where $H(z)$ is the Hubble parameter at the redshift z and H_0 is the present Hubble parameter. For an emitter at the cosmological distance $z \gtrsim 1$ we have $g_{cos}(z) \sim O(t_0^{-1})$ where t_0 is the age of universe $t_0 \sim 3 \times 10^{17}$ sec. From above equations we have $\Delta t_o = \Delta t_e(1+z) + \frac{g_{cos}(z)}{2}\Delta t_e^2 + \dots$, where Δt_o and Δt_e are the arrival time at the observer and the time at the emitter, respectively. When we observe the gravitational waves from the cosmological distance, we have $\Delta t_o = \Delta T + X(z)\Delta T^2 + \dots$, with $X(z) \equiv g_{cos}(z)/2(1+z)^2$, where $\Delta T = (1+z)\Delta t_e$ is the arrival time neglecting the cosmic acceleration/deceleration (the second term). Now for $\Delta T \sim 10^9$ sec, the time lag of the arrival time due to the cosmic acceleration/deceleration amounts to the order of second $\sim 10^{18}/(3 \times 10^{17}) \sim 1$ [sec]. From Eq. (1), if $X(z)$ is positive, then $\partial_t a(t_o) > \partial_t a(t_e)$. This clearly means that our universe is accelerating. Therefore the value of this time lag is the direct evidence for the acceleration/deceleration of the universe.

As shown in [9], if the accurate value of $X(z)$ at a single point z_s is available it is possible to determine Ω_{k0} as $\Omega_{k0} = \{1 - (dr(z_s)/dz)^2(1+z_s)^2(H_0 - 2X(z_s))^2\} \{r(z_s)^2 H_0^2\}^{-1}$, where we have assumed that the quantity $r(z) \equiv d_L(z)/(1+z)$ is obtained accurately, e.g., by SNAP. Even if the accurate values of $X(z)$ are not available for any points, we may apply the maximal likelihood method to determine Ω_{k0} . Using the value of Ω_{k0} thus determined, we can obtain the equation of state of our universe without any theoretical assumption on its matter content [9].

B Evolution of Phase of Gravitational Waves from Coalescing Binary at Cosmological Distance.—

Let us study an inspiraling compact binary system that evolves secularly by radiating gravitational wave [12]. For simplicity we study a circular orbit and evaluate the gravitational wave amplitude and the energy loss rate by Newtonian quadrupole formula. We basically follow analysis of Cutler & Flanagan [12] but properly take into account of effects of accelerating motion. The Fourier transform $\tilde{h}(f) = \int_{-\infty}^{\infty} e^{2\pi i f t} h(t) dt$ for the wave $h(t)$ is evaluated using the stationary phase approximation as $\tilde{h}(f) = K d_L(z)^{-1} M_c^{5/6} f^{-7/6} \exp[i\Phi(f)]$, where K is determined by the angular position and the orientation of the binary relative to the detector, and M_c is the chirp mass of the system. Keeping the first order term of the coefficient $X(z)$, the phase $\Phi(f)$ of the gravitational wave becomes

$$\begin{aligned} \Phi(f) &= 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}(8\pi M_{cz} f)^{-5/3} \\ &\quad - \frac{25}{32768} X(z) f^{-13/3} M_{cz}^{-10/3} \pi^{-13/3}, \end{aligned} \quad (2)$$

where t_c and ϕ_c are integral constants and $M_{cz} = M_c(1+z)$ is the redshifted chirp mass.

If we include the post-Newtonian (PN) effects up to P^{1.5}N-order, the term $3/4(8\pi M_{cz} f)^{-5/3}$ in eq.(2) should be modified as $\frac{3}{4}(8\pi M_{cz} f)^{-5/3} \left[1 + \frac{20}{9} \left(\frac{743}{336} + \frac{11\mu}{4M_c} \right) x + (4\beta - 16\pi)x^{3/2} \dots \right]$, where $x \equiv (\pi M_t f(1+z))^{2/3} = O(v^2/c^2)$ is the PN expansion parameter with M_t being the total mass of the binaries. The term proportional to β in P^{1.5}N order ($\propto x^{3/2}$) is caused by the spin effect [12,13]. In general P^NN contribution depends on the frequency f as $O(f^{(-5+2N)/3})$ and is largely different from the dependence $f^{-13/3}$ caused by the cosmic acceleration. This difference is very preferable for the actual signal analysis.

C the estimation error.—

For the circular orbit of the binary neutron stars (NSs) of mass M_1 and M_2 with the separation a at the redshift z , the frequency of the gravitational waves f is given by $f = 0.1\text{Hz}(1+z)^{-1}(M_t/2.8M_\odot)^{1/2}(a/15500\text{km})^{-3/2}$. The coalescing time t_c , the number of cycles N_{cycle} and the characteristic amplitude of the waves h_c are given by

$$\begin{aligned} t_c &= 7(1+z)(M_1/1.4M_\odot)^{-1}(M_2/1.4M_\odot)^{-1} \\ &\quad \times (M_t/2.8M_\odot)^{-1}(a/15500\text{km})^4 \text{yr} \end{aligned} \quad (3)$$

$$\begin{aligned} N_{cycle} &= 1.66 \times 10^7 (M_1/1.4M_\odot)^{-1}(M_2/1.4M_\odot)^{-1} \\ &\quad \times (M_t/2.8M_\odot)^{-1/2}(a/15500\text{km})^{5/2} \end{aligned} \quad (4)$$

$$\begin{aligned} h_c &= 1.45 \times 10^{-23} (1+z)^{5/6} (M_c/1.2M_\odot)^{5/6} \\ &\quad \times (f/0.1\text{Hz})^{-1/6} (d_L/10\text{Gpc})^{-1} \end{aligned} \quad (5)$$

Let us evaluate how accurately we can fit the parameter $X(z)$. We take six parameters $\lambda_i = \{A, M_{cz}, \mu_z, t_c, \phi_c, M_{cz}^{-10/3} X(z)\}$ in the matched filtering analysis up to 1PN-order for the phase $\Phi(f)$ and

Newtonian order for the amplitude [12]. Here A is the amplitude of signal $Kd_L(z)^{-1}M_c^{5/6}$ in the previous subsection and μ_z is the redshifted reduced mass $\mu_z = (1+z)M_1M_2/M_t$. As the chirp mass M_{cz} can be determined quite accurately, we simply put $\Delta X(z) = \Delta \{M_{cz}^{-10/3}X(z)\}/M_{cz}^{-10/3}$. For simplicity we fix the redshift of sources at $z = 1$ and calculate S/N and the error ΔX for equal mass binaries with various integration time Δt before coalescence. We use the effective factor $1/\sqrt{5}$ for reduction of antenna sensitivity due to its rotation [2]. For the present analysis we neglect the binary confusion noise since double White Dwarf binaries do not exist at frequency $f \gtrsim 0.1\text{Hz}$ [14].

We found that we can detect NS-NS binaries at $z = 1$ with $S/N \simeq 20000$ and $\Delta X/t_0^{-1} \simeq 7.0 \times 10^{-3}$ for integration time $\Delta t = 16\text{yr}$ ($N_{\text{cycle}} \sim 10^7$ orbital cycles), and $S/N \simeq 10000$ and $\Delta X/t_0^{-1} \simeq 1.26$ for $\Delta t = 1\text{yr}$ (see Fig. 2). With this detector it would be possible to determine $X(z)$ and obtain the information of the cosmic acceleration quite accurately. With $\Delta T = 16\text{yr}$ we have the estimation error for the redshifted masses as $\Delta M_{cz}/M_{cz} = 1.5 \times 10^{-11}$, $\Delta \mu_z/\mu_z = 4.2 \times 10^{-8}$ and for the wave amplitude $\Delta A/A \sim (S/N)^{-1} = 5 \times 10^{-5}$. Although the more detailed study is needed to estimate the error of the binary inclination angle, it is expected that the luminosity distance d_L can be determined accurately so that the redshift z can be determined using the inverse function $z = d_L^{-1}(\text{distance})$ of the accurate luminosity distance from *e.g.* SNAP. As a result we can know two (not redshifted) masses M_1 and M_2 for $\sim 10^5$ binaries per year up to $z = 1$ [15]. This will be large enough to establish the mass function of NS which would bring us important implications for the equation of the state of the high density matter and the explosion mechanisms of TypeII supernovae.

As the S/N and the estimation error scale as $S/N \propto h_{rms}^{-1}$ and $\Delta X \propto h_{rms}$, we can attain $\Delta X/t_0^{-1} \simeq 7.0$ for the integration time $T = 16\text{yr}$ using a less sensitive detector with $h_{rms} \sim 10^{-24}$ (1000 times worse). Even though the error bar ΔX is fairly large for this detector, the likelihood analysis would be an efficient approach to study the cosmic acceleration. Considering the estimated cosmological coalescence rate of NS-NS binaries ($\gtrsim 2 \times (10\text{Gpc}/350\text{Mpc})^3 \sim 10^5\text{yr}^{-1}$) [15], we may expect the decrease of the estimation error ΔX roughly by a factor of $\sim 1/300 = 1/\sqrt{10^5}$.

D Acceleration in the Very Early Universe.—

In the inflationary phase there was an extremely rapid acceleration of the universe. In this phase the gravitational waves were generated by quantum fluctuation [16]. With CMB quadrupole anisotropies measured by COBE, the slow-roll inflation model predicts a constraint on the stochastic background $\Omega_{GW} \lesssim 10^{-15} - 10^{-16}$ at $f \sim 0.1\text{Hz}$ [17]. Ungarelli and Vecchio [18] discussed that the strain sensitivity $h_{rms} \sim 10^{-24}$ is the required level at

$f \sim 0.1\text{Hz}$ for detecting $\Omega_{GW} \sim 10^{-16}$ by correlating two detectors for decades (see also Ref. [19]). It is important to note that the band $f > 0.1\text{Hz}$ is free from stochastic backgrounds generated by White Dwarf binaries. The radiation from neutron stars binaries is present in this band and it is indispensable to remove their contributions accurately from data stream, where effects of the cosmic acceleration would be highly important. Thus measurement of the present-day cosmic acceleration is closely related to detection of the primordial gravitational wave background that is one of the most interesting targets in cosmology. If DECIGO with $h_{rms} \sim 2 \times 10^{-27}$ at $f \sim 0.1\text{Hz}$ is available we can detect the primordial gravitational waves background even if the energy density is extremely low $\Omega_{GW} \sim 10^{-20}$ by correlating two detectors for a decade.

Confusion noise due to NS-NS (or NS-BH, BH-BH) binaries might be important in the band $f \sim 0.1\text{Hz}$. Ungarelli and Vecchio [18] investigated the critical frequency f_g where we can, in principle, remove signal from individual NS-NS binaries by matched filtering analysis and the observed window becomes transparent to the primordial stochastic background. They roughly estimated $f_g \sim 0.1\text{Hz}$ where the number of binaries per frequency bin ($\sim 10^{-8}\text{Hz}$) is less than one. But binaries around $f \sim f_g \sim 0.1$ chirp significantly within observing time scale and the situation would be more complicated than monochromatic sources [14]. Although more detailed analysis is needed, a much smaller NS-NS coalescence rate than $\sim 10^5\text{yr}^{-1}$ might be required for our analysis to be valid.

Discussions.—

The determination of the angular position of the source is crucial for matching the phase [2]. The phase modulation at the orbital radius 1AU corresponds to $2\text{AU}/c \sim 1000[\text{sec}]$. Thus, in order to match the phase within the accuracy of $0.1[\text{sec}]$ we need to determine the angular position with precision $\sim 0.1/1000 [\text{rad}] \sim 20''$. In the matched filtering analysis we can simultaneously fit parameters of the angular position as well as the relative acceleration between the source and the barycenter of the solar system. Due to their correlation in the Fisher matrix, the measured acceleration would be somewhat degraded if we cannot determine the angular position by other observational methods. Using the gravitational wave alone, we can, in advance, specify the coalescence time and the angular position of the source within some error box. If coalescence of NS-NS binaries would release the optical signal (*e.g.* Gamma Ray Bursts as proposed by [20]) we may measure the angular position accurately by pointing telescopes toward the error box at the expected coalescence time from the chirp signal. Therefore we have not tried to fit the angular position of the source in the matched filtering method [2]. We might also determine the redshift of the source by using optical

information of host galaxies.

Let us discuss the effects of the local motion g_{local} of the emitter on the second derivative d^2t_o/dt_e^2 . As the effect of bulk motion of galaxy is much smaller than cosmological effect, we estimate the internal acceleration within the galaxy based on the observational result of NS-NS binary PSR 1913+16. As shown in Table 1 of [?], the dominant contribution of its acceleration \ddot{x} comes from the global Galactic potential field and has time scale $c/\ddot{x} \sim 10t_0(R_e/10\text{kpc})(V_{rot}/200\text{kms}^{-1})$ that can be comparable to the cosmic signal g_{cos} where R_e is the effective radius of the acceleration and V_{rot} is galactic rotation velocity. However the contamination of local effect g_{local} can be reduced by taking the statistical average of many binaries as $\langle g_{cos} + g_{local} \rangle = \langle g_{cos} \rangle$. We also note that the cosmological change in phase of a coalescing binary [given by the last term in Eq. (3)] may have other applications, and may under certain circumstances be observable by the planned LISA mission.

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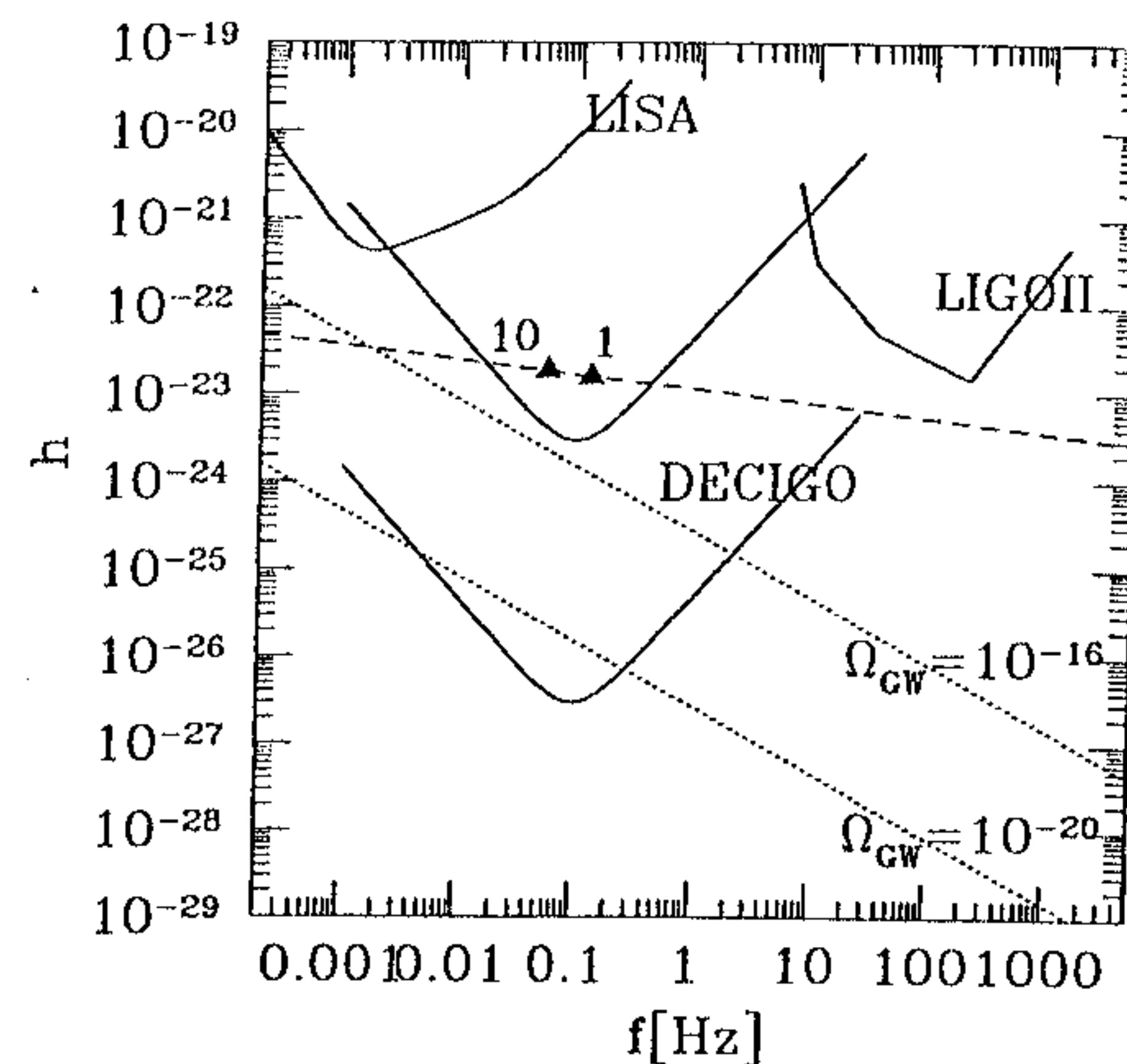


FIG. 1. Sensitivity (effectively $S/N=1$) for various detectors (LISA, DECIGO, LIGOII and a detector 10^3 times less sensitive than DECIGO) in the form of h_{rms} (solid lines). The dashed line represents evolution of the characteristic amplitude h_c for NS-NS binary at $z=1$ (filled triangles; wave frequencies at 1 yr and 10 yr before coalescence). The dotted lines represent the required sensitivity for detecting stochastic background with $\Omega_{GW} = 10^{-16}$ and $\Omega_{GW} = 10^{-20}$ by ten years correlation analysis ($S/N=1$).

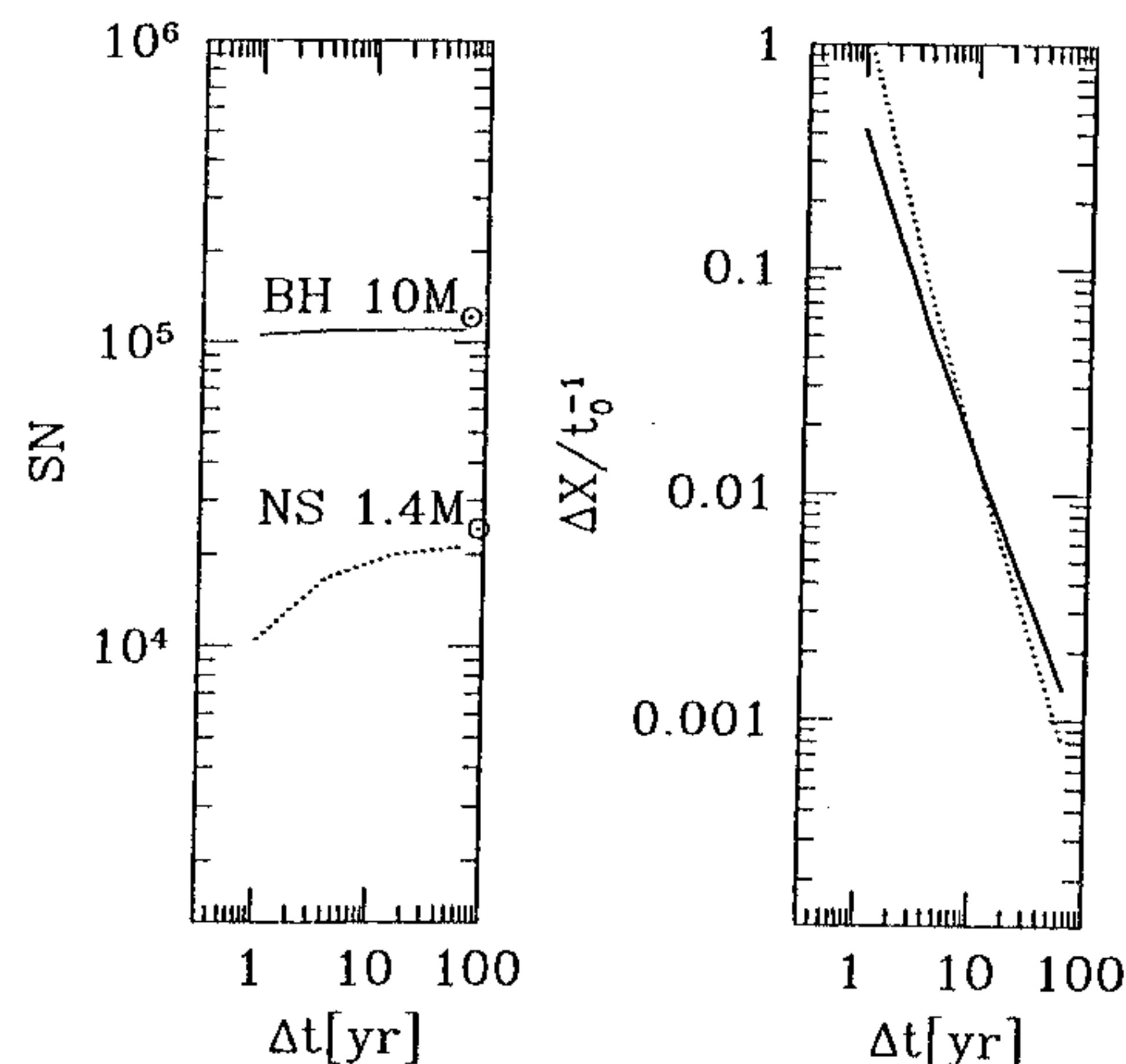


FIG. 2. S/N and the estimation error ΔX for NS-NS ($1.4M_\odot$) or BH-BH ($10M_\odot$) binaries at $z=1$. We evaluate them for DECIGO (sensitivity: $h_{rms} \sim 10^{-27}$ at $f \sim 0.1\text{Hz}$).