Qボールの安定性とダイナミクス

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Coleman's Q-Balls

- SO(2) Scalar Field

$$\mathcal{S} = \int d^4x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu \boldsymbol{\phi} \cdot \partial_\nu \boldsymbol{\phi} - V(\phi) \right]$$

$$\boldsymbol{\phi} = (\phi_1, \ \phi_2), \text{ and } \phi \equiv \sqrt{\boldsymbol{\phi} \cdot \boldsymbol{\phi}} = \sqrt{\phi_1^2 + \phi_2^2}.$$

- Conserved Charge

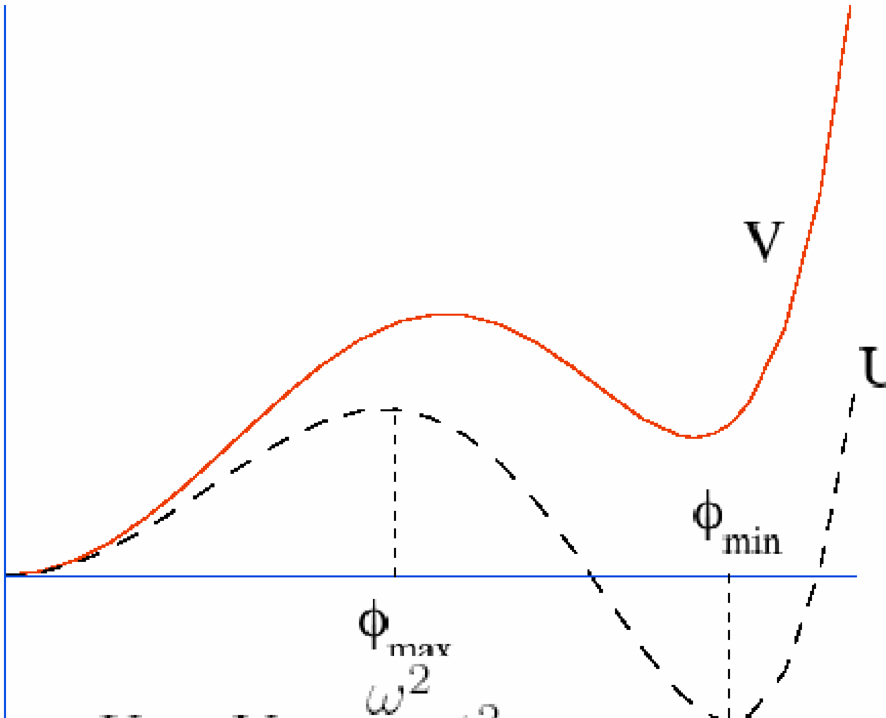
$$Q \equiv \int d^3x (\phi_1 \partial_t \phi_2 - \phi_2 \partial_t \phi_1)$$

- Assumption

$$\boldsymbol{\phi} = \phi(r)(\cos \omega t, \ \sin \omega t)$$

- Field Equation

$$\frac{d^2\phi}{dr^2} = -\frac{2}{r}\frac{d\phi}{dr} - \omega^2 \phi + \frac{dV}{d\phi}$$



$$U \equiv V - \frac{\omega^2}{2}\phi^2$$

動機と目的

- ダークマターの候補 (e.g., Kasuya & Kawasaki)
- 自己重力 を考慮した研究が殆どない。
- Qボールの平衡解、その安定性、ダイナミクスを、一般相対論的に解析。
- その前に、重力なしの場合の再解析。

(→カタストロフィー理論による安定性の解釈)

平坦な時空における平衡解

- **平衡解が存在する条件**

$$\omega_0^2 < \omega^2 < m^2, \quad \text{with} \quad \omega_0^2 \equiv \min\left(\frac{2V}{\phi^2}\right), \quad m^2 \equiv \frac{d^2V}{d\phi^2}(0)$$

具体的には、例えば

$$V = \frac{m^2}{2}\phi^2 - \lambda\phi^n + O(\phi^{n+1}), \quad m^2 > 0, \quad \lambda > 0, \quad n \geq 3$$

- **平衡解の安定性(これまでの研究)**

thin wall $(\omega^2 \to \omega_0^2)$ の極限では安定。

thick wall $(\omega^2 \to m^2)$ の極限では、$n = 3$ のとき安定

$n \geq 4$ のとき不安定

両極限以外の場合の数値解析

- Model $$V(\phi) = \frac{\phi^6}{M^2} - \lambda\phi^4 + \frac{m^2}{2}\phi^2$$

- Normalization $$\tilde{t} \equiv \lambda M t, \quad \tilde{r} \equiv \lambda M r, \quad \tilde{\phi} \equiv \frac{\phi}{\sqrt{\lambda}M}$$

$$\tilde{V} \equiv \frac{V}{\lambda^3 M^4}, \quad \tilde{U} \equiv \frac{U}{\lambda^3 M^4}, \quad \tilde{m} \equiv \frac{m}{\lambda M}, \quad \tilde{\omega} \equiv \frac{\omega}{\lambda M}$$

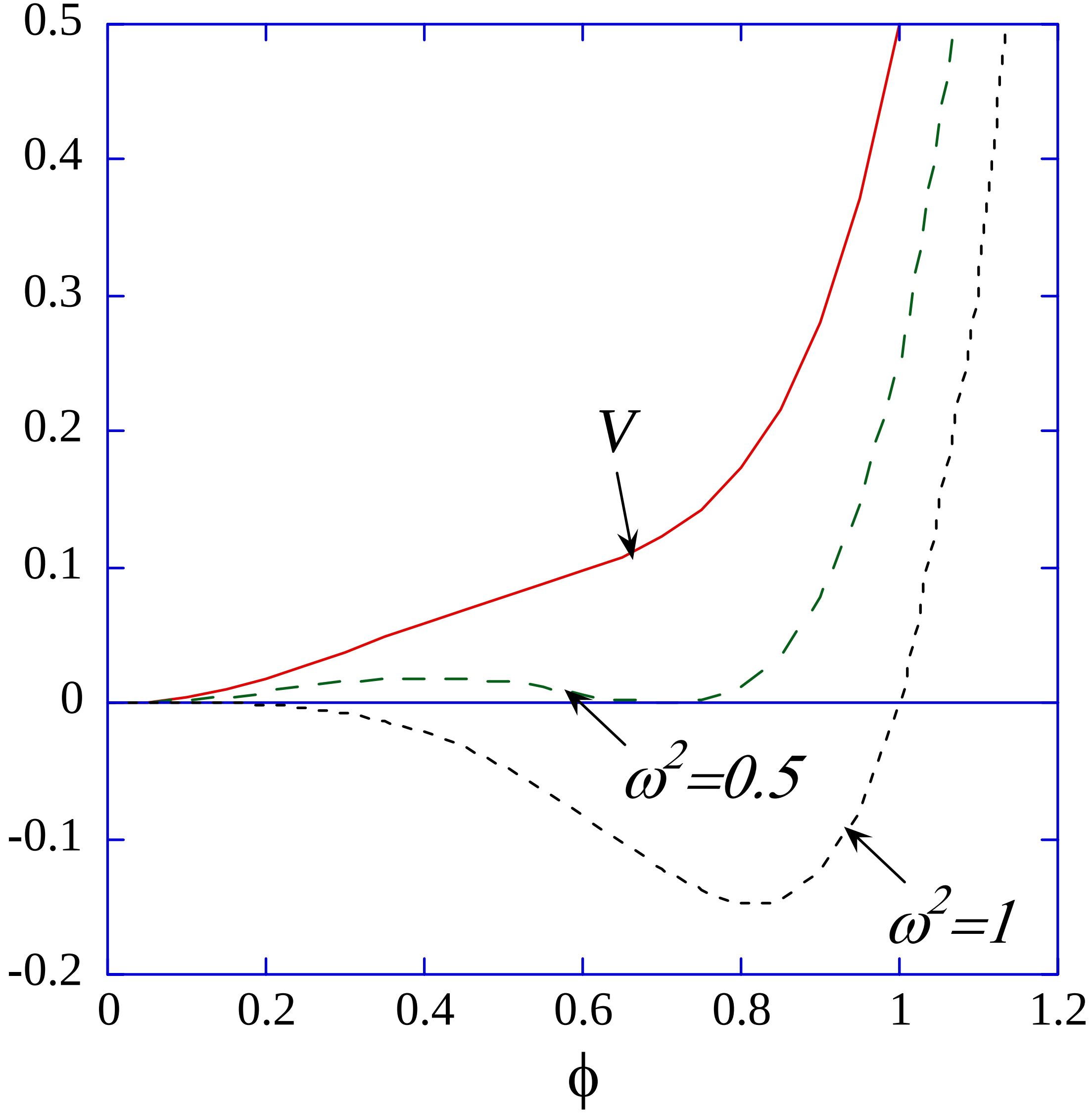
↓

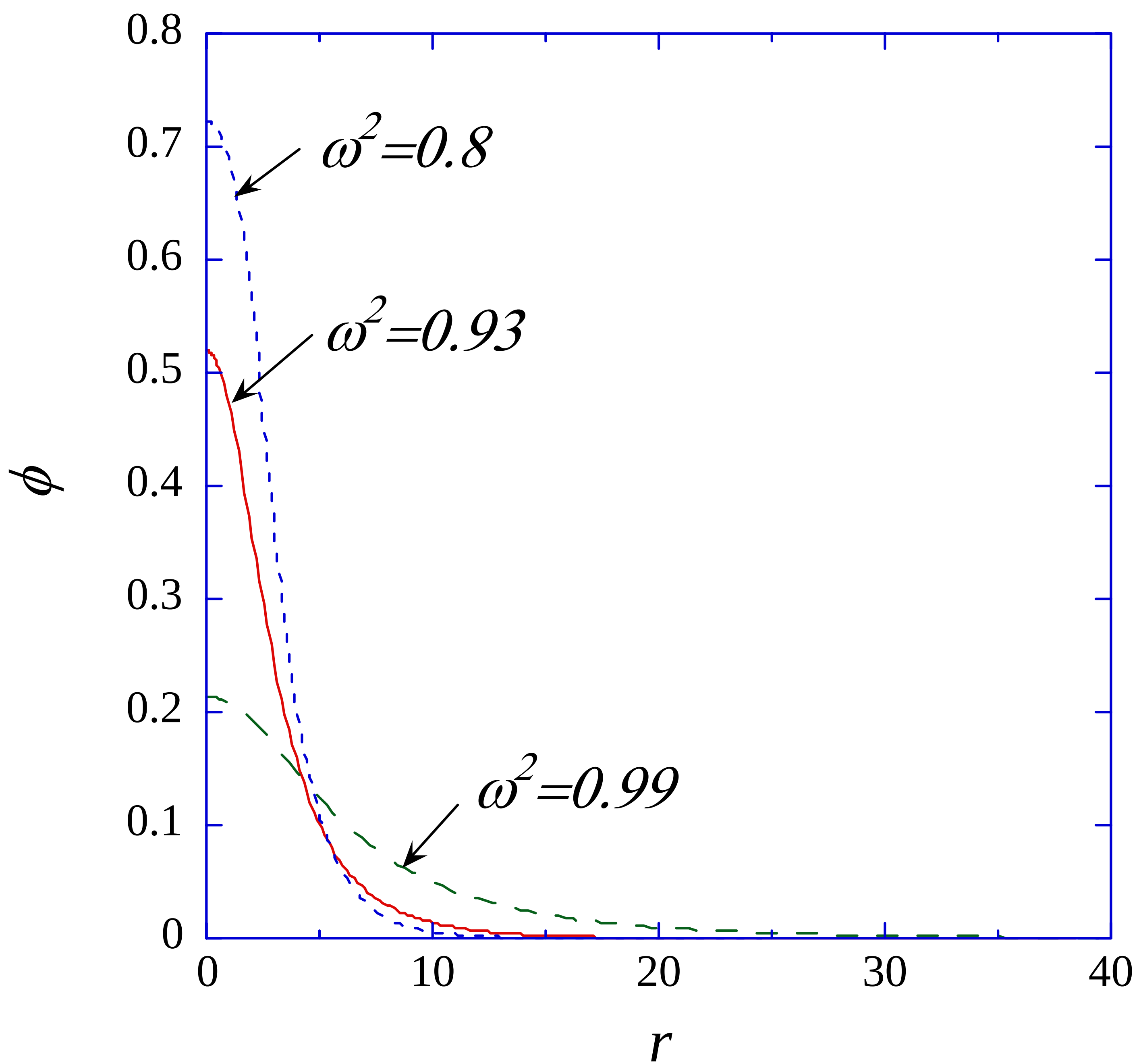
$$\tilde{V} = \tilde{\phi}^6 - \tilde{\phi}^4 + \frac{\tilde{m}^2}{2}\tilde{\phi}^2, \quad \tilde{U} = \tilde{V} - \frac{\tilde{\omega}^2}{2}\tilde{\phi}^2$$

$$\tilde{\phi}'' = -\frac{2}{\tilde{r}}\tilde{\phi}' + \frac{d\tilde{U}}{d\tilde{\phi}}$$

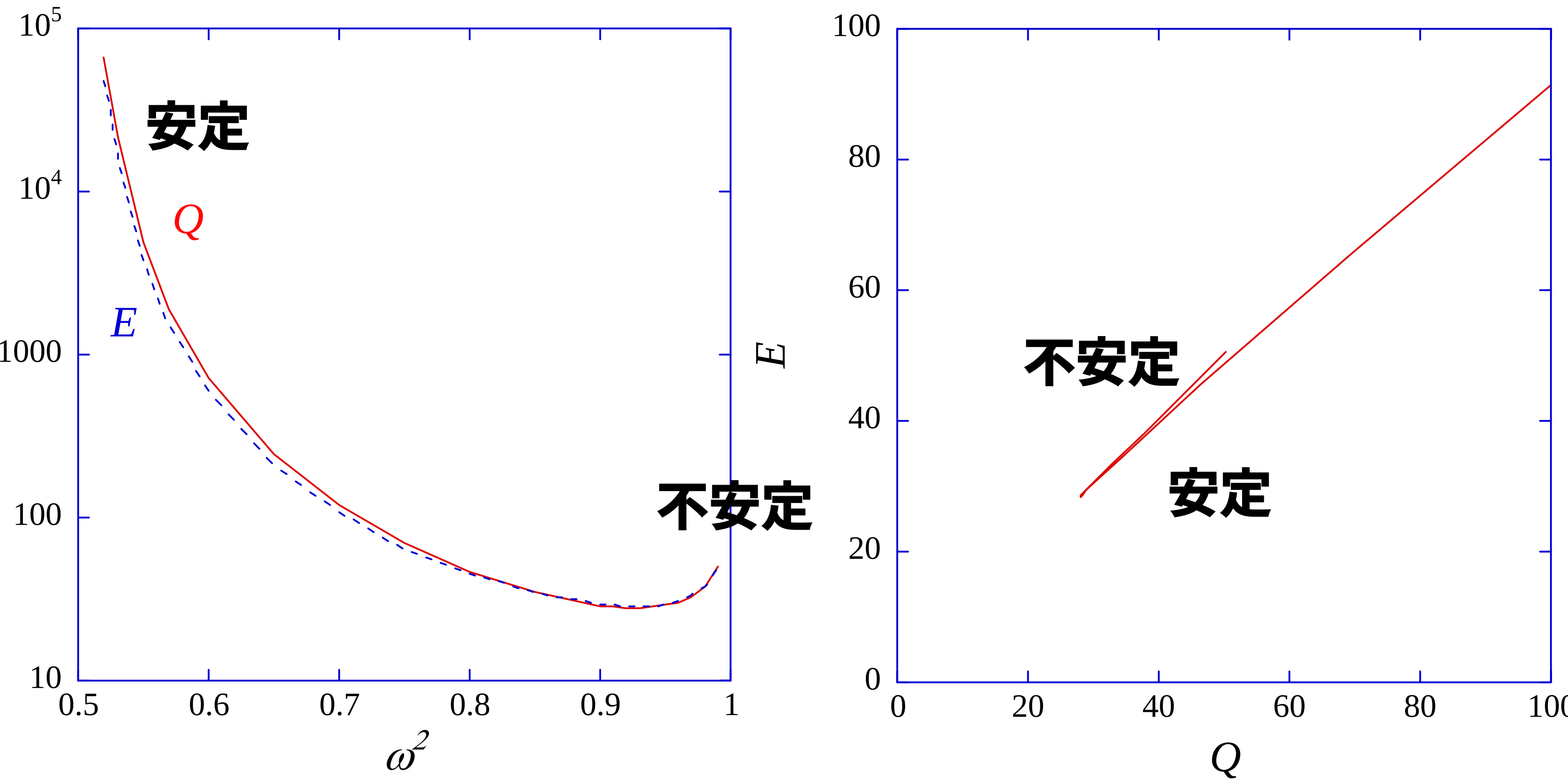
$$\tilde{V} = \tilde{\phi}^6 - \tilde{\phi}^4 + \frac{\tilde{m}^2}{2}\tilde{\phi}^2, \quad \tilde{U} = \tilde{V} - \frac{\tilde{\omega}^2}{2}\tilde{\phi}^2$$

m=1





0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
ϕ
0
10
20
30
40
r
ω²=0.8
ω²=0.93
ω²=0.99



Qをコントロールパラメーター、ωを状態変数とするカタストロフィーのタイプになっている。

線形摂動解析や数値解析と一致する。

曲がった時空における平衡解

- SO(2) scalar field + Einstein gravity

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{{m_{\mathrm{Pl}}}^2}{16\pi}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\boldsymbol{\phi}\cdot\partial_\nu\boldsymbol{\phi} - V(\phi)\right]$$

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{8\pi}{{m_{\mathrm{Pl}}}^2}\left[\partial_\mu\boldsymbol{\phi}\cdot\partial_\nu\boldsymbol{\phi} - g_{\mu\nu}\left(\frac{1}{2}g^{\lambda\sigma}\partial_\lambda\boldsymbol{\phi}\cdot\partial_\sigma\boldsymbol{\phi} + V\right)\right]$$

$$\Box\boldsymbol{\phi} = \frac{dV}{d\phi}\frac{\boldsymbol{\phi}}{\phi}$$

- Spherically symmetric and static spacetime

$$ds^2 = -\alpha^2(r)dt^2 + A^2(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Assumption of homogeneous phase rotation

$$\boldsymbol{\phi} = \phi(r)(\cos\omega t,\ \ \sin\omega t)$$

- Field equations

$$-\frac{\tilde{r}A^3}{2\lambda^2M^2}G^t_t \equiv A' + \frac{A}{2\tilde{r}}(A^2-1) = \frac{\kappa}{2}\tilde{r}A^3\left(\frac{\tilde{\phi}'^2}{2A^2} + \frac{\tilde{\omega}^2\tilde{\phi}^2}{2\alpha^2} + \tilde{V}\right),$$

$$\frac{\tilde{r}\alpha}{2\lambda^2M^2}G_{rr} \equiv \alpha' + \frac{\alpha}{2\tilde{r}}(1-A^2) = \frac{\kappa}{2}\tilde{r}\alpha A^2\left(\frac{\tilde{\phi}'^2}{2A^2} + \frac{\tilde{\omega}^2\tilde{\phi}^2}{2\alpha^2} - \tilde{V}\right)$$

$$\frac{A^2\phi}{\lambda^{5/2}M^3\phi_1}\Box\phi_1 \equiv \tilde{\phi}'' + \left(\frac{2}{\tilde{r}} + \frac{\alpha'}{\alpha} - \frac{A'}{A}\right)\tilde{\phi}' + \left(\frac{\tilde{\omega}A}{\alpha}\right)^2\tilde{\phi} = A^2\frac{d\tilde{V}}{d\tilde{\phi}}.$$

- Boundary conditions

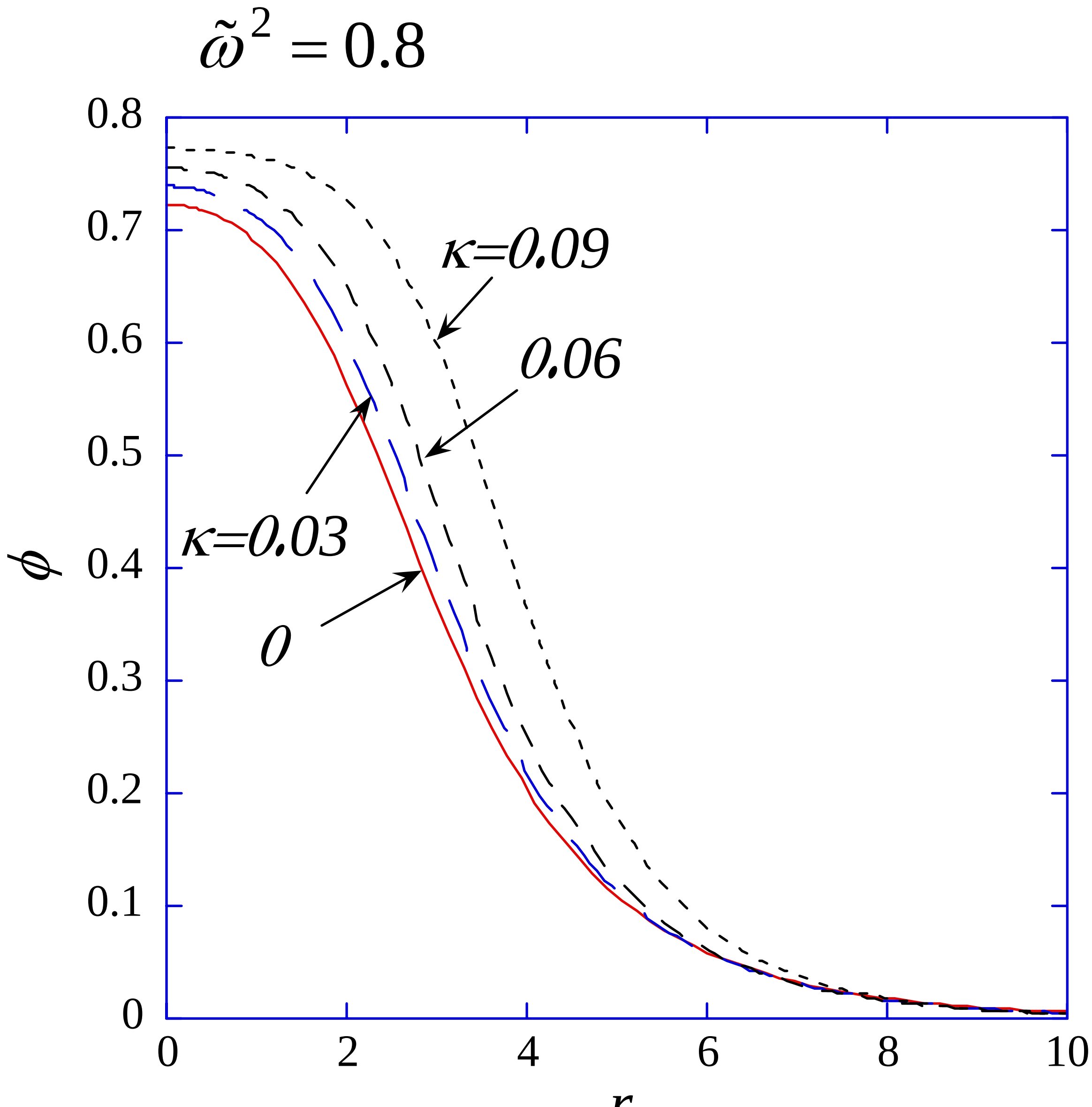
$$A=1,\quad A'=\alpha'=\tilde{\phi}'=0 \ \text{ at } \ \tilde{r}=0$$

$$A=1,\quad \tilde{\phi}=0 \ \text{ at } \ \tilde{r}\to\infty$$

- Numerical analysis

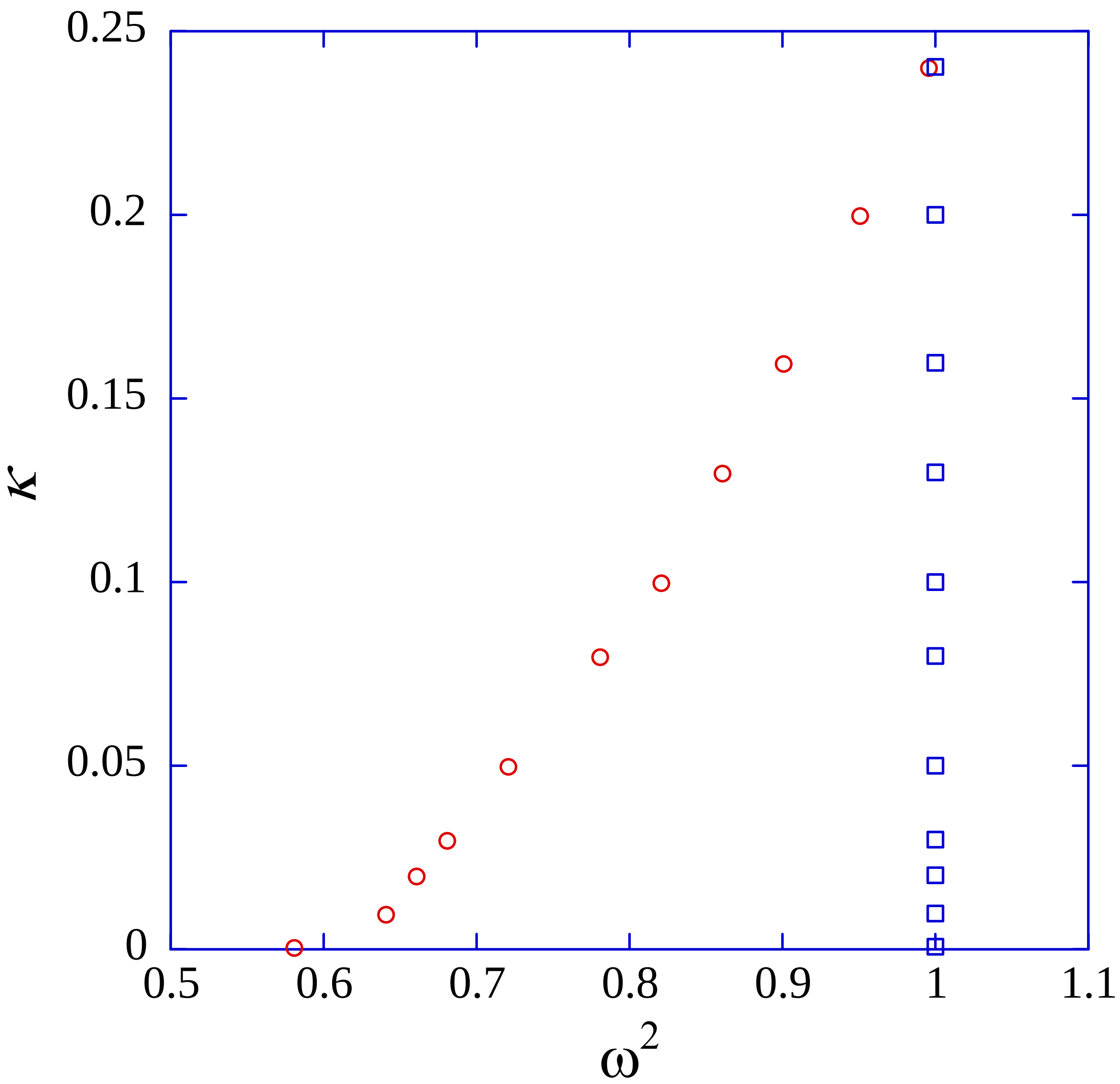
We still fix m=1.

Free parameters are **ω** and $\kappa \equiv 8\pi\lambda M^2/{m_{\rm Pl}}^2$



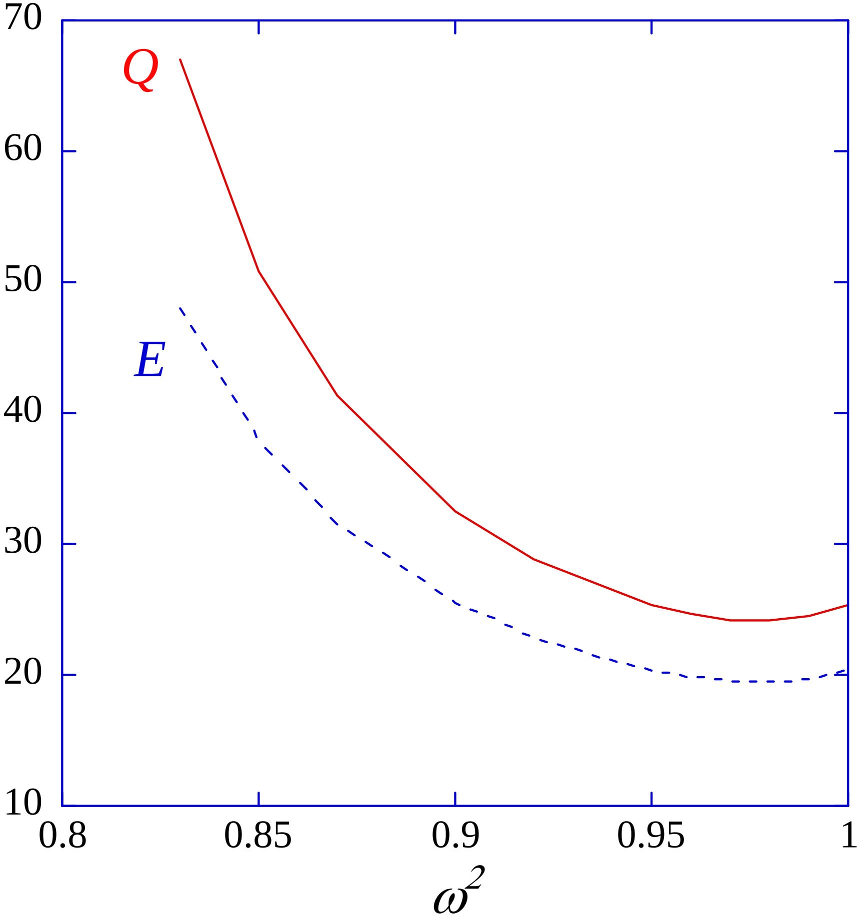
$\tilde{\omega}^2 = 0.8$
κ=0.09
0.06
κ=0.03
0
ϕ
r
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10

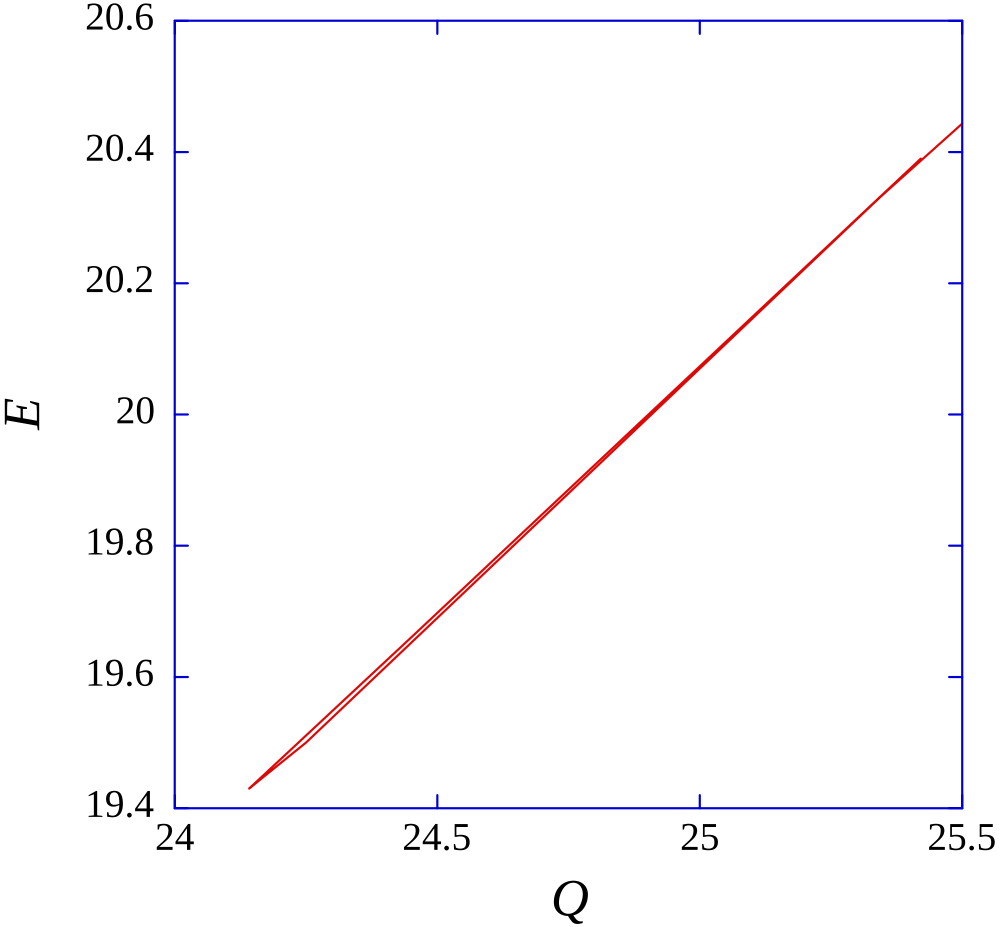
Condition for equilibrium solutions in flat spacetime.



Near Planck scale equilibrium solutions are nonexistent.

κ =0.1の定常解
Eは重力質量





Dynamical solutions in curved spacetime

- Spherically symmetric and dynamical spacetime

$$ds^2 = -\alpha^2(t,r)dt^2 + A^2(t,r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Normalization and auxiliary variables

$$\tilde{\boldsymbol{\phi}} \equiv \frac{\boldsymbol{\phi}}{\sqrt{\lambda}M} = (\tilde{\phi}_1,\ \tilde{\phi}_2), \quad \boldsymbol{\varpi} \equiv \frac{A\dot{\tilde{\boldsymbol{\phi}}}}{\alpha} = (\varpi_1,\ \varpi_2)$$

$$\boldsymbol{\xi} \equiv \partial_{\tilde{r}^2}\tilde{\boldsymbol{\phi}} = (\xi_1,\ \xi_2), \quad a \equiv \frac{A-1}{\tilde{r}^2}$$

- Field equations

$$-\frac{A^2}{\lambda^2 M^2} G^t_t \equiv \frac{4}{A}\partial_{\tilde{r}^2} A + a(1+A) = \kappa\left(\frac{\boldsymbol{\varpi}\cdot\boldsymbol{\varpi}}{2} + 2\tilde{r}^2 \boldsymbol{\xi}\cdot\boldsymbol{\xi} + A^2 \tilde{V}\right),$$

$$\frac{\tilde{r}A}{2\lambda^2 M^2} G_{tr} \equiv \dot{a} = \kappa\alpha\boldsymbol{\varpi}\cdot\boldsymbol{\xi}$$

$$\frac{\tilde{r}\alpha}{2\lambda^2 M^2} G_{rr} \equiv \alpha' - \frac{\tilde{r}\alpha a(1+A)}{2} = \frac{\kappa}{2}\tilde{r}\alpha\left(\frac{\boldsymbol{\varpi}\cdot\boldsymbol{\varpi}}{2} + 2\tilde{r}^2 \boldsymbol{\xi}\cdot\boldsymbol{\xi} - A^2 \tilde{V}\right)$$

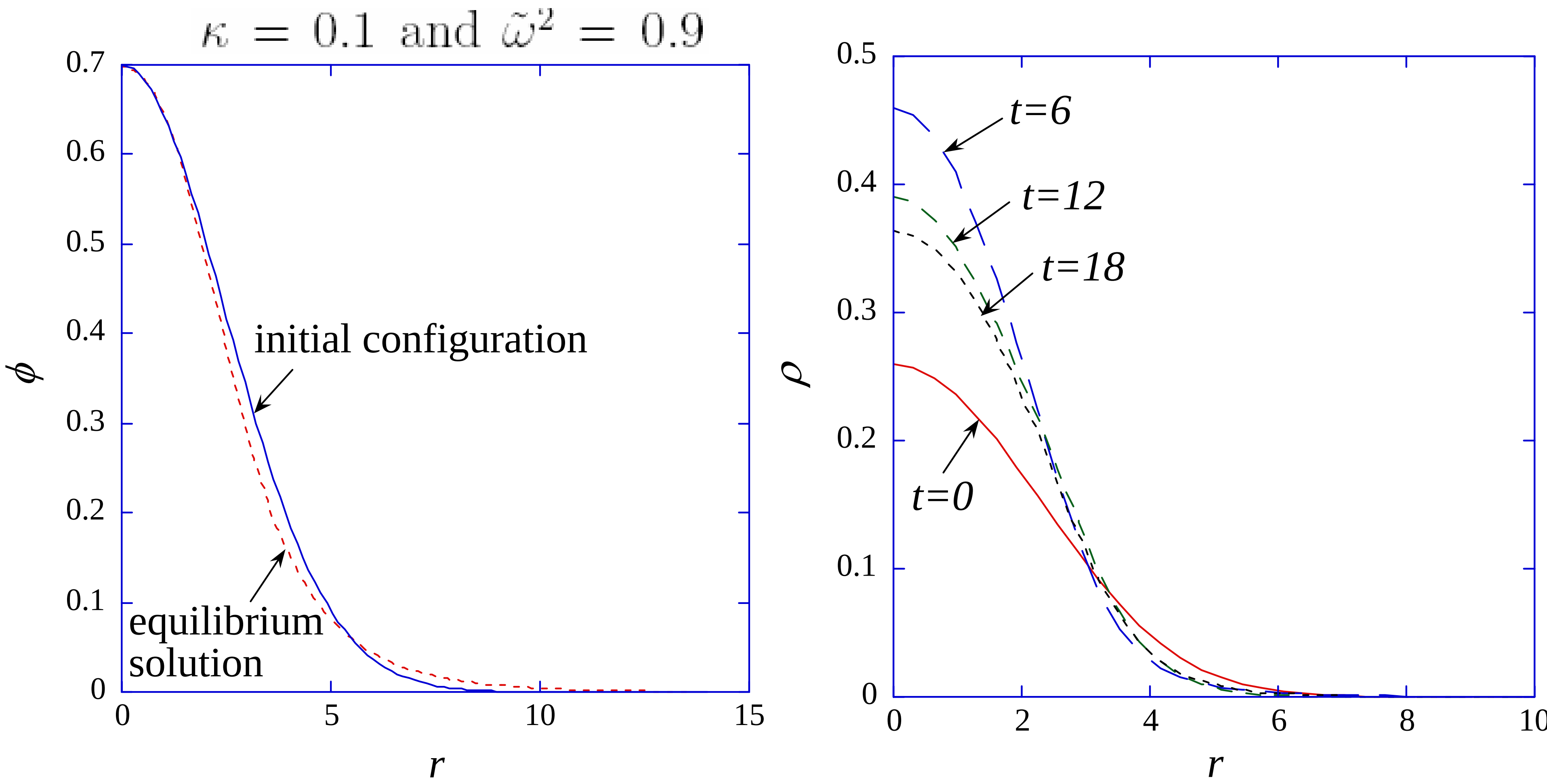
$$\frac{\alpha A}{\lambda^{5/2} M^3} \Box\tilde{\boldsymbol{\phi}} \equiv -\dot{\boldsymbol{\varpi}} + 4\tilde{r}^2 \partial_{\tilde{r}^2}\left(\frac{\alpha\boldsymbol{\xi}}{A}\right) + \frac{6\alpha\boldsymbol{\xi}}{A} = \frac{\alpha A\tilde{\boldsymbol{\phi}}}{\tilde{\phi}}\frac{d\tilde{V}}{d\tilde{\phi}}$$

- Initial conditions

$$\tilde{\boldsymbol{\phi}} = (f(\tilde{r}),\ 0) \quad \dot{\tilde{\boldsymbol{\phi}}} = (0,\ \tilde{\omega} f(\tilde{r}))$$

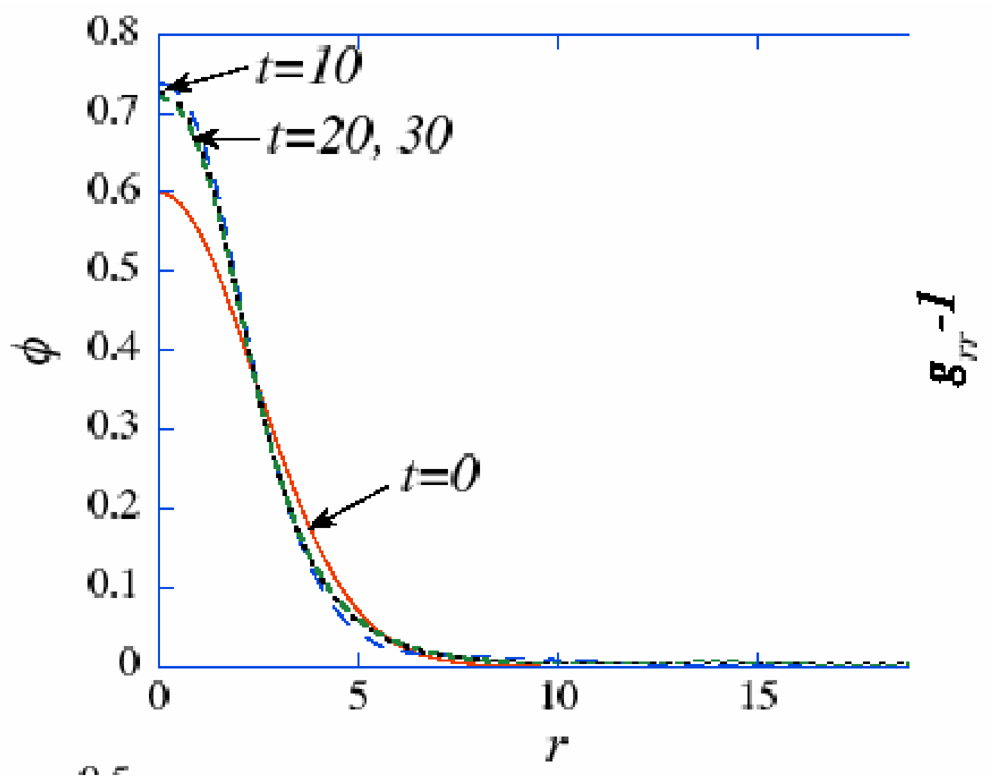
$$f(\tilde{r}) = C\exp\left[-\left(\frac{\tilde{r}}{\tilde{r}_0}\right)^2\right]$$

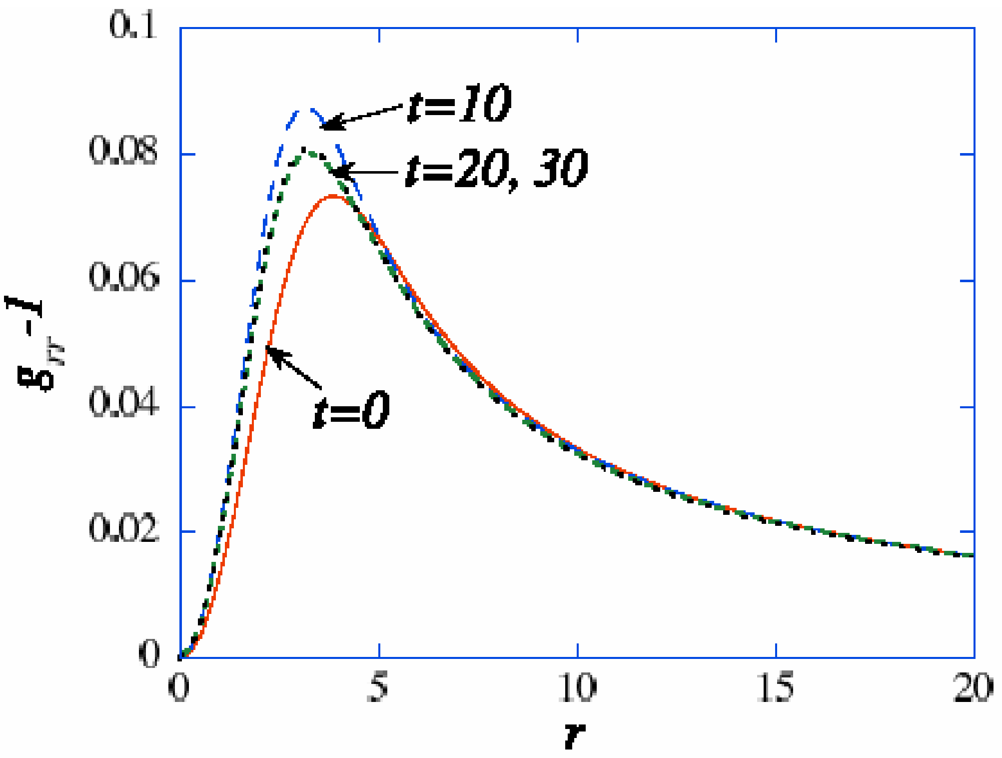
Perturbations on equilibrium solutions



If **κ** >0.3, equilibrium solutison are nonexistent.
What happens?

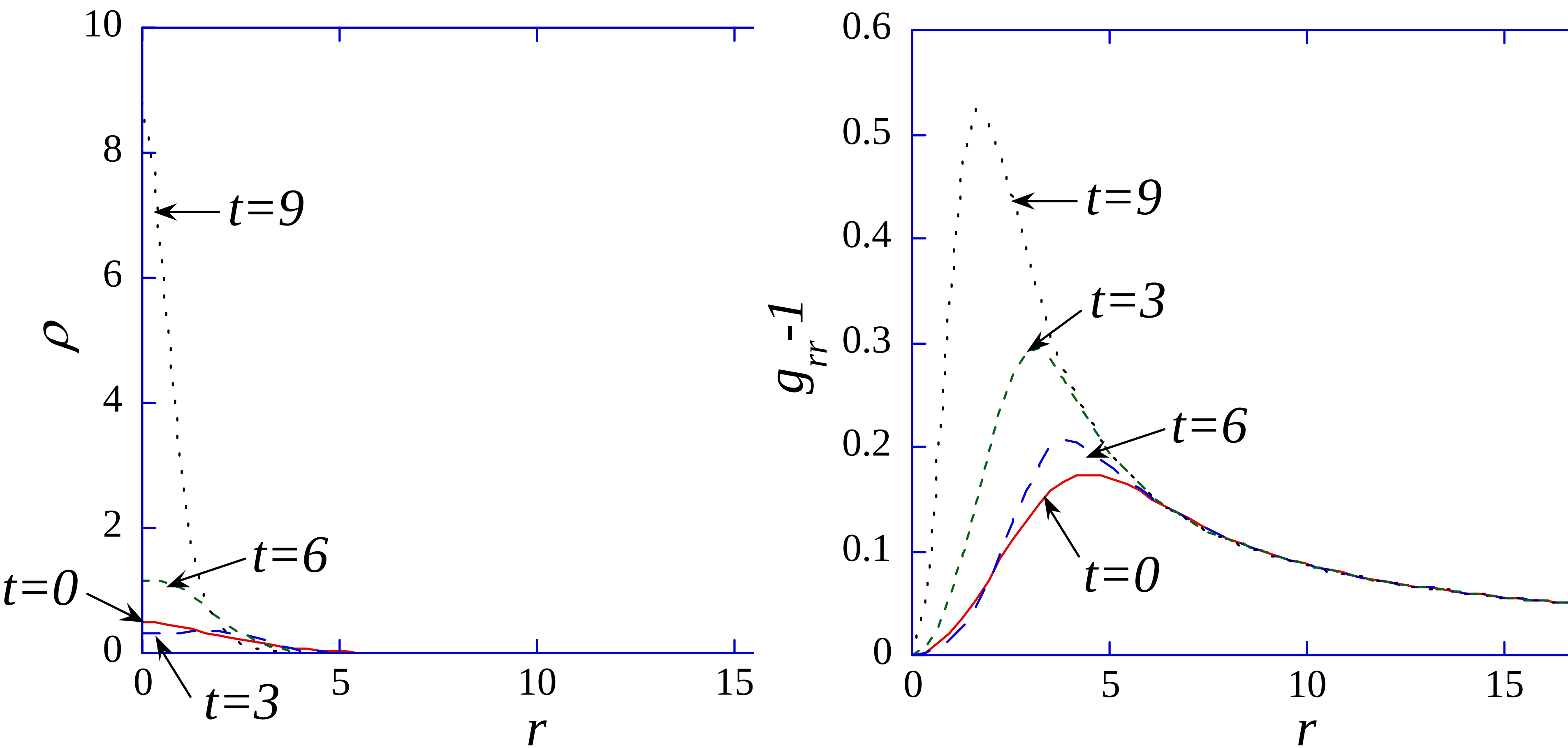
$\kappa = 0.3$ and $\tilde{\omega}^2 = 1$





Both **Φ** and metric approaches to stable configurations.

For different initial condition with larger Q



Black hole formation.

まとめ

- 重力の有無によらず、平衡解のω、Q、Eの関係に注目すると、カタストロフィー理論によってその安定性を理解することができる。
- 重力の効果が大きくなると、平衡解存在するωの範囲が小さくなる。
- プランクスケールに近くなると、平衡解が存在しなくなる。初期条件によって、ブラックホールになる場合と、準定常解（？）になる場合がある。