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# The Effect of Poloidal Magnetic Field on Planetary Migration

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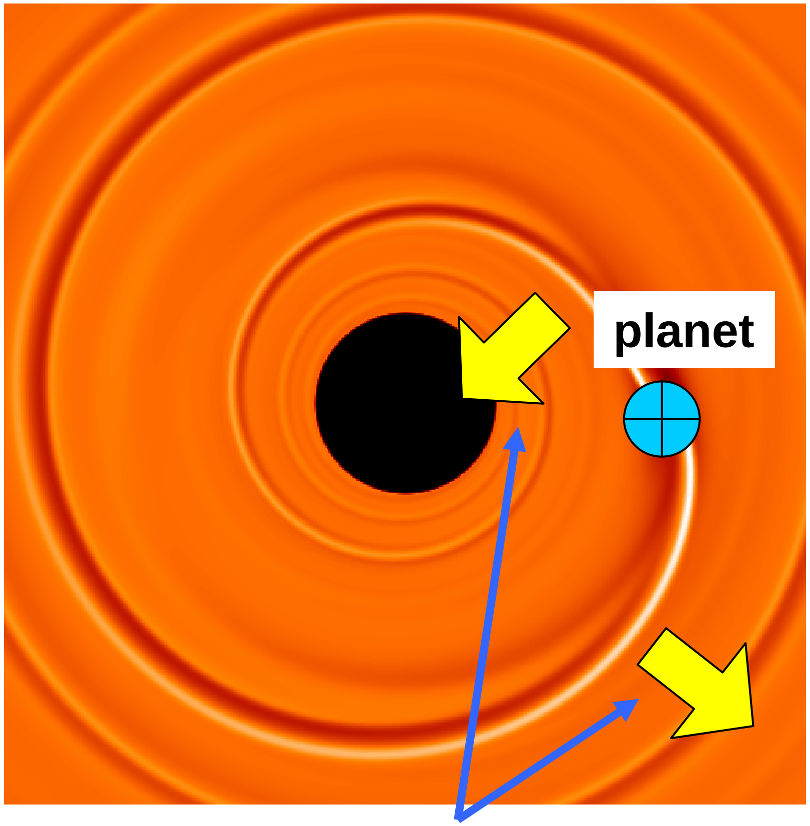
*Planetary migration is the radial motion of protoplanets due to the gravitational interaction between protoplanets and circumstellar gas. Recent work has shown that when a protoplanet of an Earth mass is formed at 1AU, it falls towards the central star before the dispersal of the protoplanetary disk.*

*In order for protoplanets to survive, some mechanism should act to stop the infall. Recently, it has been shown that toroidal magnetic field may stop the inward planetary migration under a certain condition, so magnetic field should be important. In this poster, the effect of weak poloidal magnetic field is investigated.*

# (Type I) Planetary Migration




**Angular momentum transfer**

- Protoplanets generate ***density waves*** on protoplanetary disk
- Waves transport angular momenta
- Outer wave wins
- The planets ***fall towards the central star*** due to the ***backreaction*** of the waves

From Dr F Masset's web page

http://www.maths.qmul.ac.uk/~masset/moviesmpegs.html

# Timescale of Planetary Migration

Tanaka, Takeuchi and Ward ApJ **565** 1257 (2002)

$$\tau = (2.7 + 1.1\alpha)^{-1} \frac{M_c}{M_p} \frac{M_c}{\sigma_p r_p^2} \left(\frac{c}{r_p \Omega_p}\right)^2 \Omega_p^{-1}$$

$\sigma \propto r^{-\alpha}$ :surface density  $c$ :thermal velocity (isothermal)

$M_c$ :central star mass  $M_p$ :planet mass

Minimum Mass Solar Nebula: $\sigma = 150(r/5\mathrm{AU})^{-3/2}\ \mathrm{g\ cm^{-2}}$

$T = 130\ \mathrm{K\ at\ 5\ AU}$

For $M_c = M_\odot \quad M_p = M_\oplus$ at 5AU

$$\boxed{\tau \sim 8 \times 10^5 \mathrm{yr} < \tau_{nebula} \sim 10^7 \mathrm{yr}}$$

**Protoplanets Fall into Central Star *BEFORE* Gas Dispersal**

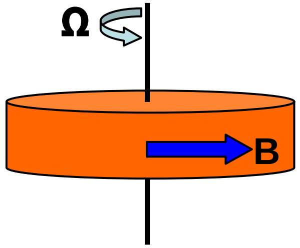
# Stopping Inward Planetary Migration by …

Terquem MNRAS **341** 1157 (2003)

**Toroidal magnetic field** can strengthen the wave inside the planet orbit.

1. ***Magnetic Resonance*** appears
2. When magnetic field decreases slower than

$$B \propto r^{-1}$$



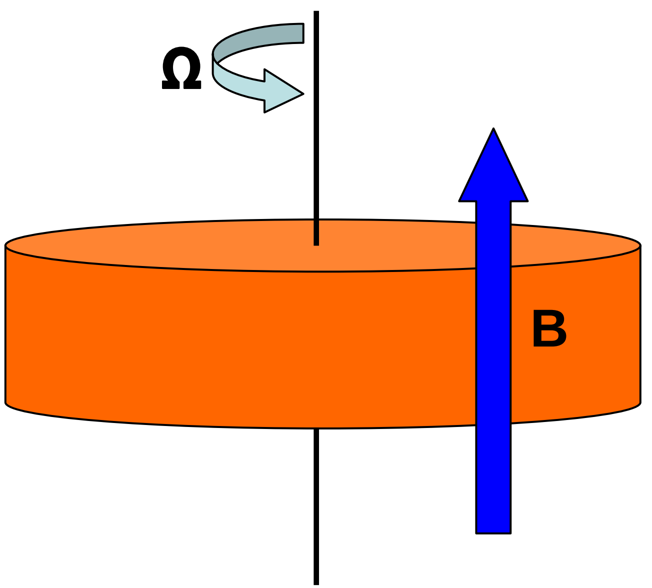

planets migrate ***outward***

Magnetic Field may be Important when Considering Planetary Migration

# Ongoing Research

Muto and Inutsuka in prep

- Terquem considered toroidal field
- Do ***poloidal fields*** do any good?




- Polodal fields usually originate outside the disk
  - Always exist
- Need considering disk thickness
- Any change in wave propagation on the disk?
- Relation with MRI?

# Basic Equations (1)

Ideal MHD+One Planet's Gravitational Force

Background: $\mathbf{v}_0 = (0, r\Omega(r), 0)$ $\mathbf{B}_0 = (0, 0, B_0(r))$

No planet in background

No structure in z direction, but disk exists only at -H<z<H

H: Scale height of the disk

Neglect disk self-gravity

Fourier expand the perturbation

$\rho \longrightarrow \rho_0 + \delta\rho$ : linearisation

$$\delta\rho \rightarrow \delta\rho(r)\exp[-i(\omega t - m\phi/r - k_z z)]$$

# Basic Equations (2)

Consider stationary perturbation: $\omega \rightarrow m\Omega_p + i\gamma$

gamma: small viscosity is taken into account if necessary

Isothermal perturbation: $c^2 = \dfrac{d\delta p}{d\delta\rho}$

Local approximation: $d/dr, m/r, k_z \gg 1/r$

Lagrangian displacement: $\delta v_r = -i\sigma\xi_r \quad \delta v_z = -i\sigma\xi_z$

$$\delta v_\phi = -i\sigma\xi_\phi - \xi_r r \frac{d\Omega}{dr}$$

with $\sigma = \omega - m\Omega(r)$

# Perturbation Equations

EoC: $$\frac{\delta\rho}{\rho_0} + \frac{d\xi_r}{dr} + \frac{im}{r}\xi_\phi + ik_z\xi_z = 0$$

EoM (r component):

$$-c^2\frac{d}{dr}\frac{\delta\rho}{\rho_0} + \left(\sigma^2 + 4A\Omega - k_z^2 v_A^2\right)\xi_r + v_A^2\frac{d^2\xi_r}{dr^2} - 2i\Omega\sigma\xi_\phi + v_A^2\frac{im}{r}\frac{d\xi_\phi}{dr} = -\frac{GM_p}{r_p}\frac{dg}{dr}$$

EoM (phi component):

$$-c^2\frac{im}{r}\frac{\delta\rho}{\rho_0} + 2i\Omega\sigma\xi_r + v_A^2\frac{im}{r}\frac{d\xi_r}{dr} + \left(\sigma^2 - v_A^2\left(k_z^2 + \frac{m^2}{r^2}\right)\right)\xi_\phi = -\frac{im}{r}\frac{GM_p}{r_p}g$$

EoM (z component): $$-c^2 ik_z\frac{\delta\rho}{\rho_0} + \sigma^2\xi_z = -ik_z^2\frac{GM_p}{r_p}g$$

with $$g(r) \sim \frac{2}{\pi}\int_0^\pi d\phi\frac{\cos(m\phi)}{\sqrt{1 - 2(r/r_p) + (r/r_p)^2 + \epsilon}} \equiv b_{1/2}^m(r,\epsilon)$$

# Torque Exerted on the Disk

Once density perturbation is known,
torque exerted on the disk is calculated:

$$\boxed{T = -\int r dr d\phi dz \rho (\mathbf{r} \times \nabla \psi_p(\mathbf{r})) \cdot \mathbf{e}_z}$$

$$T_{m,k_z} = 2\pi r_p^2 G M_p \rho m h \int dx g(x) \Im \left( \frac{\delta\rho}{\rho} \right)$$

with $x = (r - r_c)/r_c$

Torque on the planet is back reaction

# Equation Governing the Density Perturbation

$$\left[\frac{d^2}{dr^2} + \left(\frac{d}{dr}\ln\frac{\sigma^2 - v_A^2 k_z^2}{D}\right)\frac{d}{dr} + \frac{(\sigma^2 - c^2 k_z^2)D}{(\sigma^2 - v_A^2 k_z^2)((c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2)} - \frac{m^2}{r^2}\frac{(\sigma^2 - v_A^2 k_z^2 + 8A\Omega)D - 16A\Omega\sigma^2\left(\sigma^2 - v_A^2 k_z^2 - \frac{1}{2}\kappa^2\right)}{(\sigma^2 - v_A^2 k_z^2)D}\right] f = - \frac{\sigma^2 D}{(\sigma^2 - v_A^2 k_z^2)((c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2)}\frac{GM_p}{r_p} g$$

$$D = (\sigma^2 - v_A^2 k_z^2)(\sigma^2 - v_A^2 k_z^2 - \kappa^2) - 4\Omega^2 v_A^2 k_z^2$$

$$f = \frac{(c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2}{\sigma^2}\frac{\delta\rho}{\rho_0} - \frac{\sigma^2 - v_A^2 k_z^2}{\sigma^2}\frac{GM_p}{r_p} g$$

Basically, this equation is of the form:

$$\boxed{\frac{d^2 f}{dr^2} + \mathcal{A}_1 \frac{df}{dr} + \mathcal{A}_0 f = \mathcal{S}}$$

# Wave Propagation on Disk

WKB Approximation: $d/dr, k_z \gg m/r$

but $m^2(c^2+v_A^2)/(r^2\Omega^2) \ll 1$

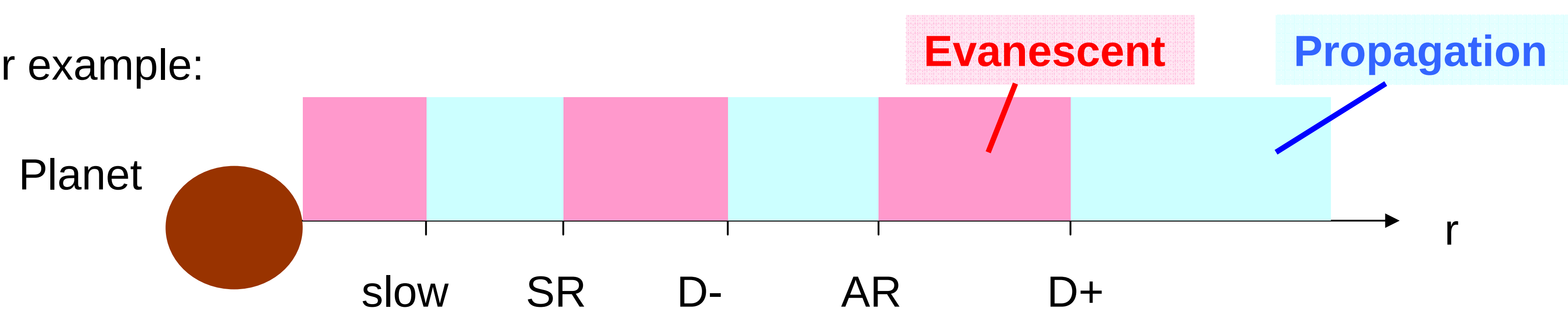
$$\boxed{\left[\frac{d^2}{dr^2}+\frac{(\sigma^2-c^2k_z^2)D}{(\sigma^2-v_A^2k_z^2)((c^2+v_A^2)\sigma^2-c^2v_A^2k_z^2)}\right]f=-\frac{\sigma^2 D}{(\sigma^2-c^2k_z^2)((c^2+v_A^2)\sigma^2-c^2v_A^2k_z^2)}\frac{GM_p}{r_p}g}$$

Lindblad resonance: $D=0$ or $\sigma^2 = v_A^2k_z^2 + \frac{\kappa^2 \pm \sqrt{\kappa^4+16\Omega^2 v_A^2 k_z^2}}{2}$

Alfven resonance: $\sigma^2 = v_A^2 k_z^2$ Sound resonance: $\sigma^2 = c^2 k_z^2$

Slow resonance: $\sigma^2 = c^2 v_A^2 k_z^2/(c^2+v_A^2)$

For example:

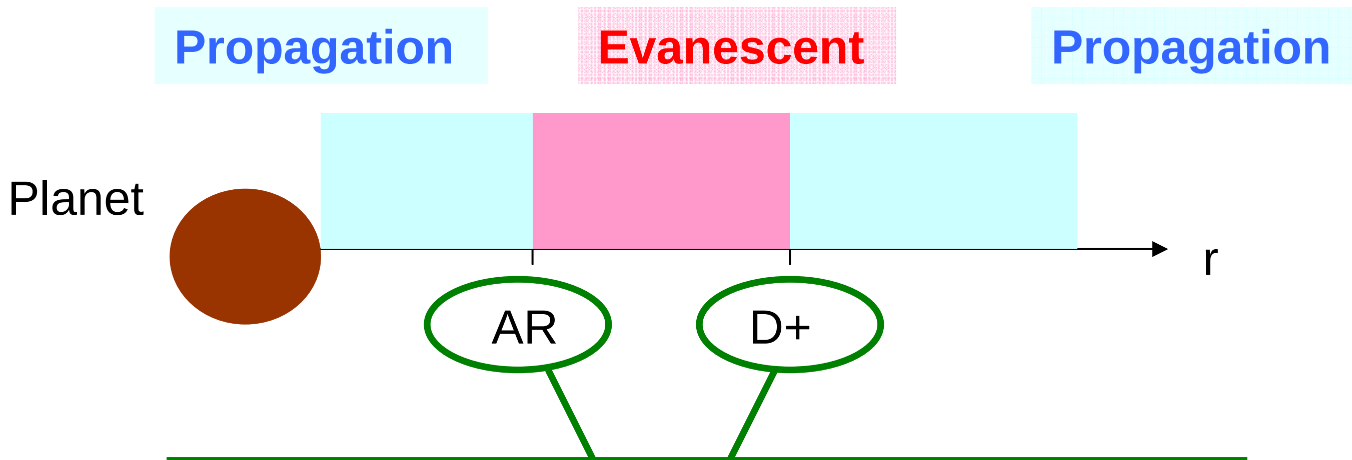


# c=0, weak B Case

For simplicity, we consider:

$c = 0$ and $v_A^2 k_z^2 \ll 4\Omega A$

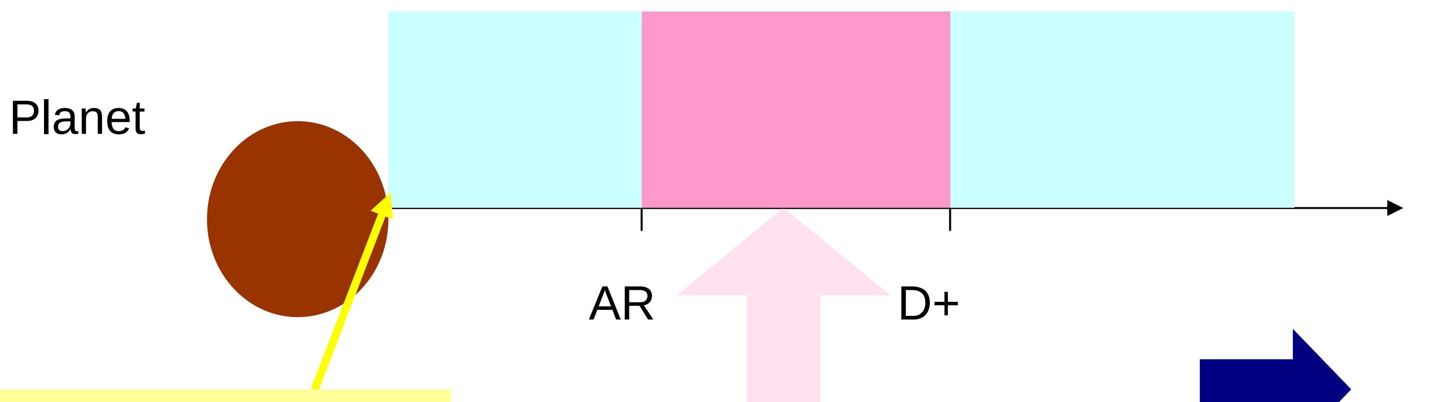
Rem: disk is MRI unstable




Solve the wave equation with appropriate boundary conditions in the vicinity of resonances

# Boundary Conditions

**Propagation** | **Evanescent** | **Propagation**




**Outgoing wave**

consistency of **outgoing boundary** and the **symmetry** of the equation

$$\Im \tilde{f}(\tilde{x}=0)=0$$

$$\Re \tilde{f}'(\tilde{x}=0)=0$$

**Waves should not grow exponentially**

Rem: this poster focuses on Lindblad resonance

# Lindblad Torque Formula

$$\boxed{T_{m,k_z} = -m\pi^2 GM_p(2\rho h)\frac{\kappa^2 + \sqrt{\kappa^4 + 16\Omega^2 v_A^2 k_z^2}}{8m\sigma_{LR}A\sqrt{\kappa^4 + 16\Omega^2 v_A^2 k_z^2}} v_p^2 \left[\frac{dg}{dx} - \frac{2m\Omega\sigma}{\sigma^2 - v_A^2 k_z^2} g\right]_{LR}^2}$$

This formula ***generalises*** Goldreich and Tremaine formula

$$\boxed{T_{m,k_z} = -m\pi^2 GM_p(2\rho h)\frac{1}{4m\sigma_{LR}A} v_p^2 \left[\frac{dg}{dx} - \frac{2m\Omega}{\sigma} g\right]_{LR}^2}$$

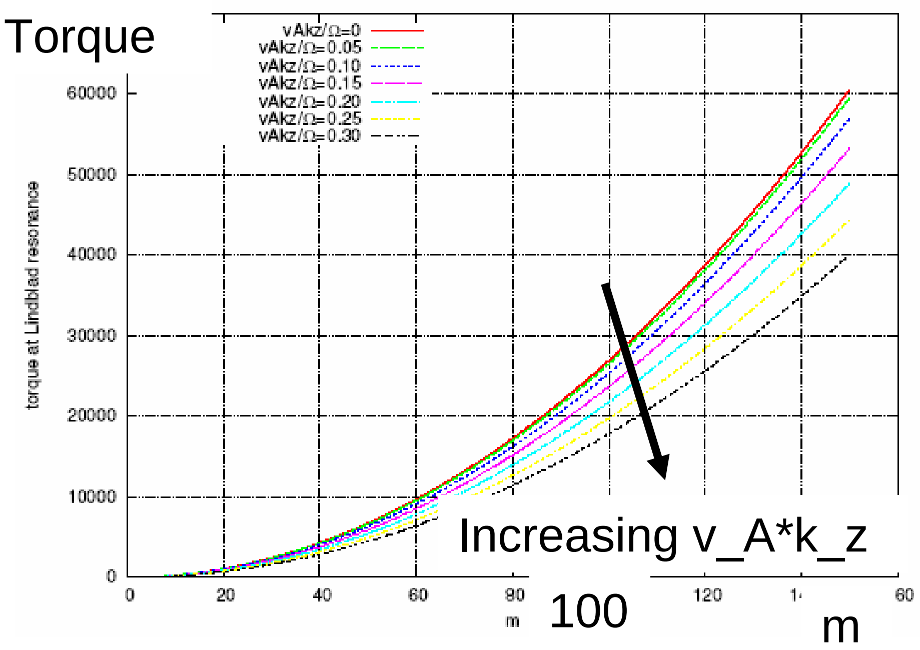
Goldreich and Tremaine *ApJ* **233** 857 (1979)

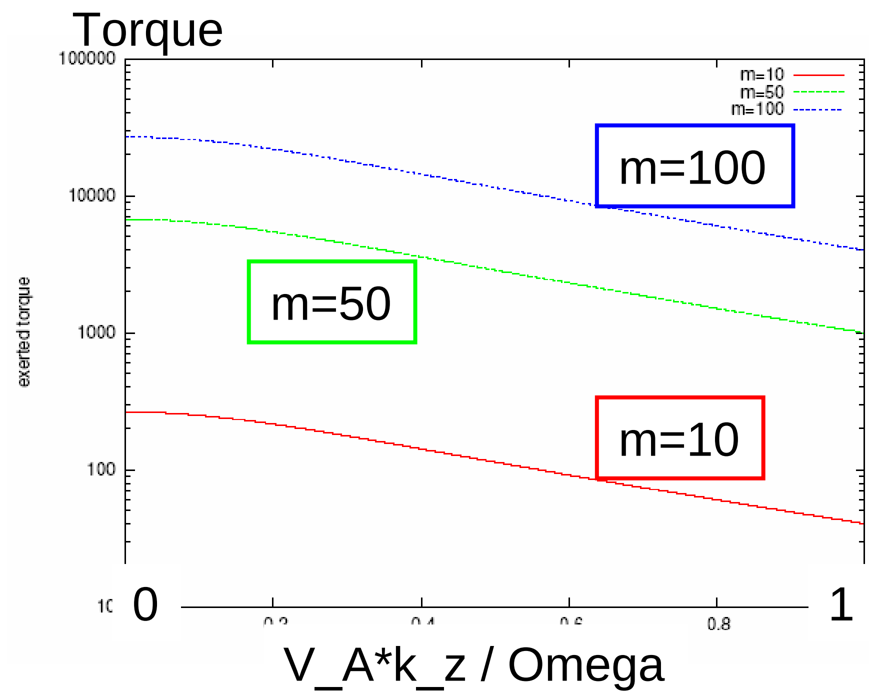
'… the torque exerted at an isolated resonance is independent of particular physics at work …'

Meyer-Vernet and Sicardy *Icarus* **69** 157 (1987)

# Discussion: How Torque Affected?

Lindblad resonance is **further apart from planet**




Torque is **suppressed** by about **a factor of 5**

Migration timescale is **5 times longer**

# Summary

- We considered the effect of ***poloidal magnetic field*** on planetary migration.
- Under WKB approximation, we derived an ***analytic torque formula*** at Lindblad resonance when sound speed is negligible.
- Lindblad torque is ***suppressed*** in the presence of magnetic field.
- More detailed analysis is under way.