Torsional Oscillations of Relativistic Stars with Dipole Magnetic Fields

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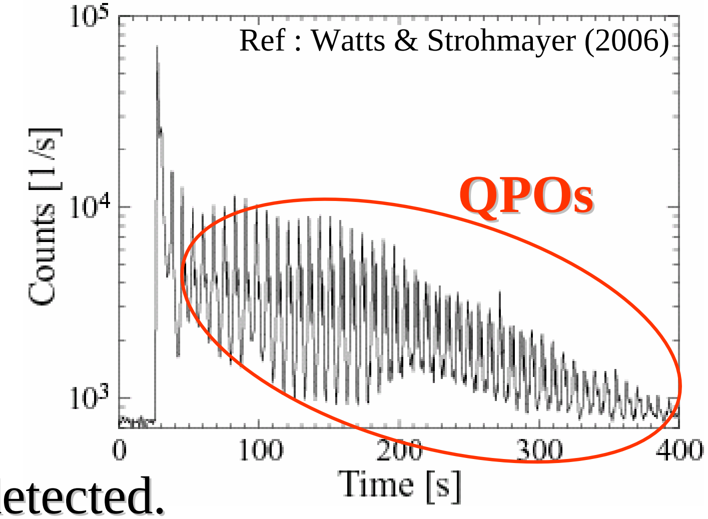
QPOs in SGRs

- SGRs
  - The most promising model : magnetar (Duncan & Thompson 1992)

  → the magnetic stress in the solid crust

  → the gamma-ray burst



- Giant flares in SGRs
  - Up to now, three giant flares have been detected.
    - SGR 0526–66 in 1979, SGR 1900+14 in 1998, SGR 1086–20 in 2004
  - Peak luminosities : 10^{44} – 10^{46} erg/s
  - A decaying tail for several hundred seconds follows the flare.
- QPOs in decaying tail (Israel *et al*. 2005; Watts & Strohmayer 2005, 2006)
  - SGR 1900+14 : 28, 54, 84, and 155 Hz
  - SGR 1086–20 : 18, 26, 29, 92.5, 150, 626.5, and 1837 Hz

    (possible additional frequencies : 720 and 2384 Hz)

How to explain QPOs

- QPOs is due to the torsional oscillation in crust ??
  - In Newtonian; Hansen & Cioffi (1980), McDermott *et al*. (1998), Carroll *et al*. (1986), Storhmayer (1991), …
  - In GR; Schumaker & Thone (1983), Leins (1994), Messions *et al*. (2001), Samuelsson & Andersson (2006)

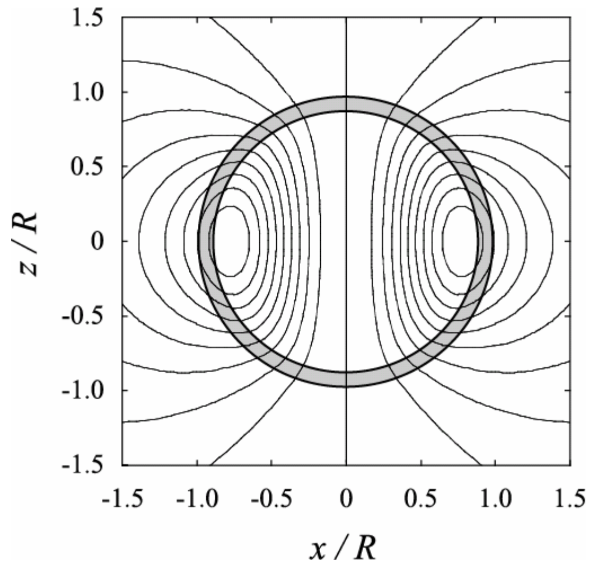
→ without magnetic field or non-realistic one !!

- QPOs can be explained by global Alfven modes ??
  - Glampedakis *et al*. (2006)
    - simple toy model → the possibility to explain the observational data !!
- With dipole magnetic field & realistic EOS
  - Crust torsional modes
  - Global Alfven modes

  ⟺ Observational frequencies

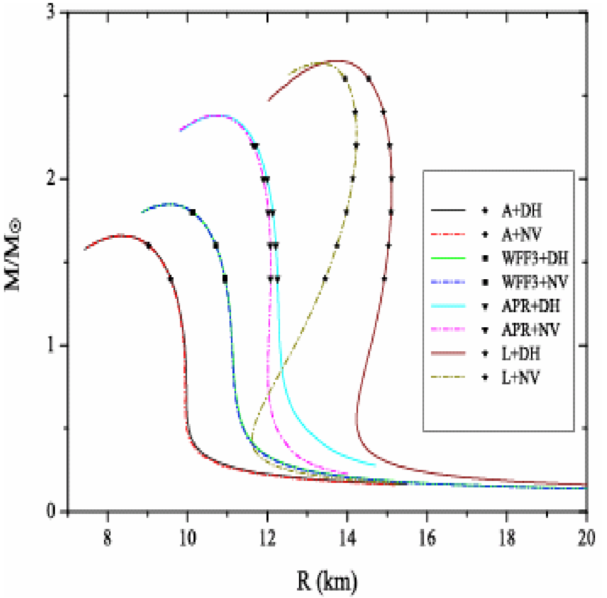
Models of Magnetar

- Ideal MHD approximation
  → electric field is zero for comoving observer.
- Neglect the deformation due to the magnetic tension
  - Magnetic energy / gravitational energy
    $\sim 10^{-4}$ (B/10^{16}[G])
  - Equilibrium configuratin :
    static and spherically symmetric one.
- Axisymmetric poloidal magnetic field
- In the crust : isotropic shear modulus **μ**
  → Shear modulus depends on the EOS of crust.



EOS

- Inner core : four EOS such as A, WFF3, APR, and L
  - EOS A is a very soft and EOS L is a very stiff.
- Crust : two EOS such as NV and DH
  - NV : Negele & Vautherin (1973)
    $\rho \sim 2.4 \times 10^{14}$ [g/cc] at the basis of crust
    $\mu \sim 1.267 \times 10^{30} \rho_{14}^{4/5}$ [erg/cc]
  - DH : Douchin & Haensel (2001)
    $\rho \sim 1.28 \times 10^{14}$ [g/cc] at the basis of crust
    $\mu \sim 1.74 \times 10^{30} \rho_{14}^{3/4} (Z/53)^2 (616/A)^{4/3}$
    $\times ((1-X_n)/0.41)^{4/3}$ [erg/cc]



Perturbations

- Linearized the equation of motion and Maxwell's equations
  - Axisymmetric axial perturbation
  - Cowling approximation ($\delta g_{\mu\nu} = 0$)
- Perturbation equation to be solved

→ eigenvalue equation of 1-dimensional form !

$$\ell(\ell+1)\mathcal{A}_\ell + \mathcal{L}_1^{\pm 2}\mathcal{B}_\ell + \mathcal{L}_2^{\pm 2}\mathcal{C}_\ell = 0$$

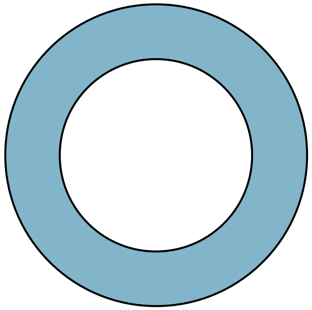
→ the ℓ-th order perturbation is coupled with the ($\ell \pm 2$)-th order.

For simplicity, we neglect these coupling.

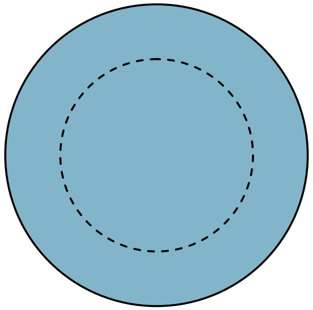
→ We need to consider the case including coupling terms later…

Boundary Conditions

- For pure crust torsional modes;
  - Zero traction condition at basis of crust and at stellar surface.
    → at basis of crust, it is not true boundary condition with magnetic field !
    → in the limit of non-magnetic field, it is correct one.
    → thus for weak magnetic field this B.C. is not so bad.
- For global Alfven modes;
  - Regularity condition at stellar center
  - Zero traction condition at stellar surface
  - Traction is continuous at basis of crust



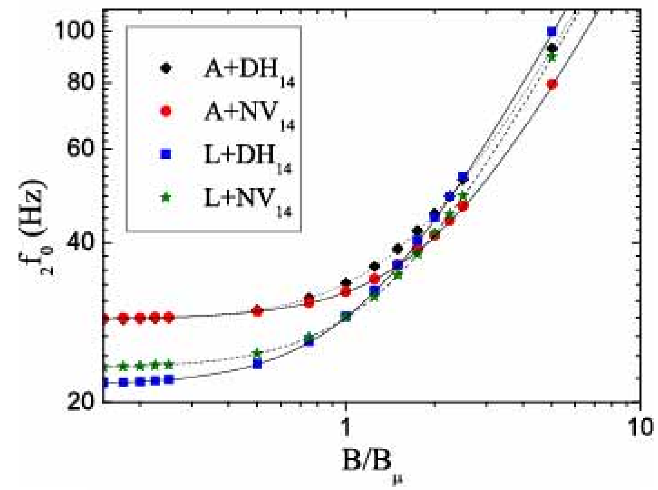
pure crust torsional modes



global Alfven modes

Pure Crust Torsional Modes

- Non-magnetized case
  - For fundamental modes
    - Frequencies depend on the stellar parameters.
      → it can vary by up to 30 ~ 50 %
    - The choice of the crust EOS does not affect significantly (1~5%)



  - For overtones
    - Frequencies are practically independent of the harmonic index ℓ.
    - The variations in the frequencies due to different choices of both the high-density and crust EOS are significant.
      → ex) frequencies of first overtones vary the range of 500 – 1200 Hz.
- The effect of the magnetic fields
  - We derive the empirical formula such as $\frac{{}_\ell f_n}{{}_\ell f_n^{(0)}} \sim \left[1 + {}_\ell\alpha_n \left(\frac{B}{B_\mu}\right)^2\right]^{1/2}$, where $B_\mu = 4 \times 10^{15}$ G.
  - For $B > B_\mu$, the shift in the frequencies is significant.

Attempt to fit to Observational data

- Some of the stiff models fit quite well to the observational data
  - SGR 1900+14 : L+NV$_{25}$ (28, 54, 84, 155 Hz → ${}_3t_0$, ${}_6t_0$, ${}_9t_0$, ${}_{17}t_0$)
  - SGR 1806–20 : L+DH$_{17}$ or L+NV$_{20}$
    (18, 29, 92.5, 150, 626.5, 1837 Hz → ${}_2t_0$, ${}_3t_0$, ${}_9t_0$, ${}_{15}t_0$, ${}_lt_1$, ${}_lt_4$)
- However …
  - it is difficult to explain the observational data of 26 Hz, because this data is very close to the other data of 29 Hz.
  - Similarly, the spacing between the 626.5 and 720 Hz in SGR 1806–20 may be too small to be explained by consecutive overtones of crust torsional modes.

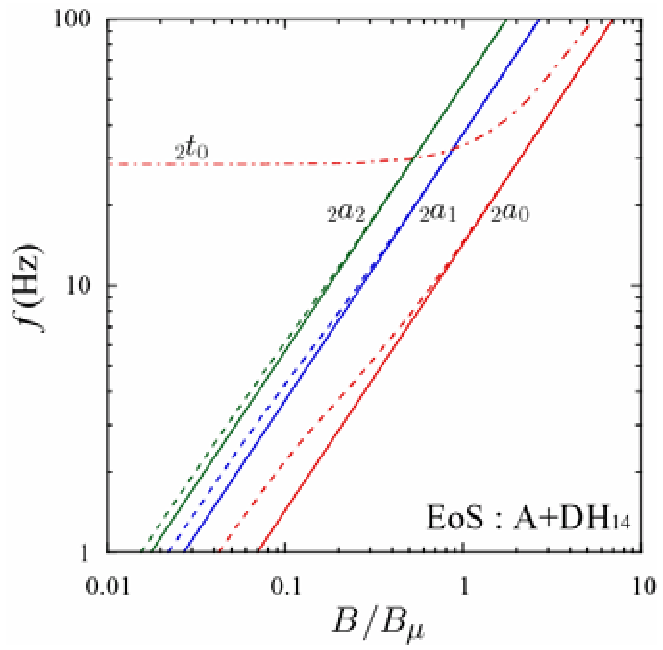
→ it may be difficult to explain all frequencies by only crust torsional modes!!

→ **It may be necessary to consider the global Alfven modes !!**

Global Alfven Modes

- Realistic stellar models have very thin solid crust.
  → the frequencies of global modes approaches that of the pure Alfven modes.
- For weak magnetic field,
  - The presence of solid crust has some effect on the global Alfven modes
- For strong magnetic field,
  - The effect diminishes !!

  → The global modes for strong magnetic field are almost identical to pure Alfven modes.
- The frequency of overtones depend on the value of ℓ !!



Comparison with Observational data

- All observational data can be explained by global Alfven modes !

  → Owing to the non-degenerate of overtones for different values of ℓ.

  Ex) SGR 1806–20 : $B/B_\mu \sim 1.25$
  SGR 1900+14 : $B/B_\mu \sim 1.94$
  for A+DH_{14} stellar model.

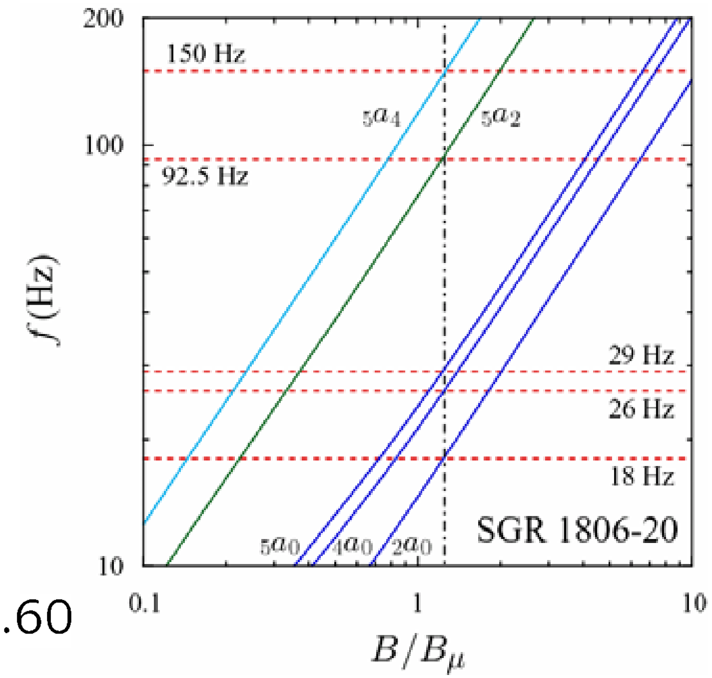
  → for other stellar models it is easy to fit !!

  - Of course, this magnetic field is strong.

  → it is easier to fit for the weaker magnetic fields, because there exist more modes.

  Ex) SGR 1806–20 : L+NV_{18} with $B/B_\mu \sim 0.60$

  → ${}_4a_0$, ${}_8a_0$, ${}_{10}a_0$, ${}_2a_6$, and ${}_2a_{11}$ for $f < 150$ Hz



- The maximum magnetic fields exist !!
  - (0.8 ~ 1.2) x 10^{16} [G]

Conclusion

- QPOs in SGRs
- Some observational data can be explained by crust torsional modes.
  → However, it is difficult to explain all observational data !!
- The overtones of global Alfven modes do not degenerate with ℓ.
  → In order to explain the observational data, this is good feature!
- With global Alfven modes, it is possible to explain the all observational data.
- The existence of maximum magnetic field.
  - $(0.8 \sim 1.2) \times 10^{16}$ [G]

Future Works

- Including the $(\ell \pm 2)$-th coupling terms
- Non-axisymmetric perturbation
- Different distribution of back-ground magnetic fields
- Polar perturbation
- Without Cowling approximation
- With the effect of deformation due to the magnetic pressure