Can thick braneworlds be self-consistent?

2005年度第18回理論懇シンポジウム
2005年12月25日(日) - 27日(火)
京都大学基礎物理学研究所

Masato Minamitsuji (Osaka/YITP)
Wade Naylor (RITS) & Misao Sasaki (YITP)

(hep-th/0508093, to appear in NPB & hep-th/0510117, to appear in PLB)

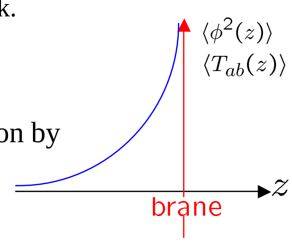
Introduction

Quantum effects of a bulk scalar field in Randall-Sundrum (RS) braneworld

$$S \sim \int d^5x\sqrt{-g}\left(\overset{(5)}{R} - (\partial\phi)^2 - 2V(\phi) - \sigma(\phi)\delta(z)\right)$$

Issues about surface divergences

Quantum effects of KK modes suffer divergences as one approaches the brane from the bulk.



How do we regularize them?

Now we propose a new regularization by

a finite brane *thickness* .

Issues of this work

: Possibility of regularization of the surface divergences

: Theoretical bound on the brane thickness parameter.

A thick de Sitter brane model

A de Sitter domain wall solution supported by χ.

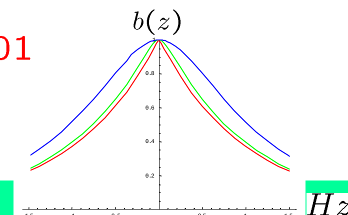
$$S_{\rm wall} = \frac{1}{2}\int d^{d+1}x\sqrt{-g}\left(\overset{(d+1)}{R} - (\partial\chi)^2 - 2V_0\left(\cos\left[\frac{\chi}{\chi_0}\right]\right)^{1-\sigma}\right)$$

$$ds^2 = b^2(z)\left(dz^2 + \gamma^{(\rm dS)}_{\mu\nu}dx^\mu dx^\nu\right), \quad \chi = \chi(z)$$

$$b(z) = \left(\cosh\left(\frac{Hz}{\sigma}\right)\right)^{-\sigma}, \quad H^2 = \frac{2\sigma V_0}{(d-1)\left[1+(d-1)\sigma\right]},$$

$$\chi(z) = \sqrt{(d-1)\sigma(1-\sigma)}\,\sin^{-1}\left(\tanh\left(\frac{Hz}{\sigma}\right)\right).$$

$0 < \sigma < 1$: brane thickness



Quantum fluctuations

Quantized scalar field perturbations

$$S = S_{\rm wall} + \frac{1}{2}\int d^{d+1}x\sqrt{-g}\phi\left(\Box_{d+1} - \xi\overset{(d+1)}{R}\right)\phi$$

$$\phi(z,x^\mu) = \sum_n\left(a_n u_n(z,x^\mu) + a_n^\dagger u_n^*(z,x^\mu)\right),$$

$$\left[a_n, a_{n'}^\dagger\right] = \delta_{n,n'}.$$

$$\langle\phi^2(z,x^\mu)\rangle = \sum_n |u_n(z,x^\mu)|^2 \qquad \partial_z\phi(z)|_{z=0} = 0,$$

Mass spectrum: $\frac{m_n^2}{H^2} = q_n^2 + \frac{(d-1)^2}{4}$

$q_0 = i\frac{\sqrt{1+4(\xi_c-\xi)(d(d-1)\sigma^2+2d\sigma)}-1}{2\sigma}$; Bound state

$q_n > 0, \quad n = 1, 2, 3, \cdots$; KK modes

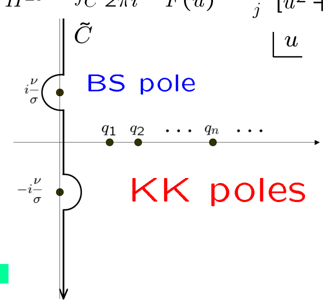
Work in Euclideanized space: $\mathrm{R}\times S^d$

Integrate over volume of S^d

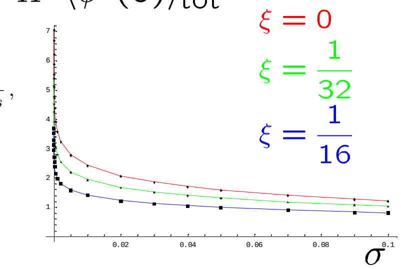
E.g., 2D dS brane: $d_j = 2j+1$

Zeta function $\langle\tilde{\phi}^2(z)\rangle_{\rm tot} = \lim_{s\to 1}\tilde{\zeta}(z,s)$

$$\tilde{\zeta}(z,s) = \frac{2\mu^{2(s-1)}}{H^{2s}}\oint_{\tilde{C}}\frac{du}{2\pi i}\frac{\sigma u G(u,z)}{F(u)}\sum_j\frac{d_j}{[u^2+(j+\frac{1}{2})^2]^s},$$



$H^2\langle\tilde{\phi}^2(0)\rangle_{\rm tot}$



Quantum fluctuations regularized.

Model independent feature

Quantum Backreaction

$$T_{ab}(z,x^i) = \phi_{;a}\phi_{;b} - \frac{1}{2}g_{ab}\phi^{;c}\phi_{;c},$$
$$x^i = (\theta,\varphi)$$

Untwisted config. $f_n^{(+)}(-z) = f_n^{(+)}(z)$

Twisted config. $f_n^{(-)}(-z) = -f_n^{(-)}(z)$

$$T^{\pm}{}_{ab}(z,x^i;s) = \frac{1}{2}\lim_{X'\to X}\left(\partial_a\partial_{b'} - \frac{1}{2}g_{ab}g^{cd}\partial_c\partial_{d'}\right) \times \zeta^{\pm}(z,x^i,z',x^{i\prime};s)$$

Zeta function regularization

Then, twisted + untwisted

(Brane) self-consistency

$$G^a{}_b|_{z=0} = \left(\overset{(0)}{T}{}^a{}_b + \langle T^a{}_b\rangle\right)_{z=0}$$

$$|\overset{(0)}{T}{}^a{}_b| \gg |\langle T^a{}_b\rangle|$$

Bound on brane thickness

$$\sigma \gtrsim (H/M_5)$$

4-dimensional dS brane

Consistency with $H\ell \gg \sigma$

$$M_{pl} \gg M_5$$

$\because \ell = M_{\rm pl}^2/M_5^3$ (in RS)

Summary & Future Work

Summary

The finite thickness of the brane can *naturally* regularize the quantum fluctuations.

Roughly speaking, self-consistency on the brane requires $M_{Pl} \gg M_5$; *Natural !*

Future work related to these issues

Quantum effects in a co-dim 2 braneworld.

For KK gravitons (i.e., spin 2 fields)