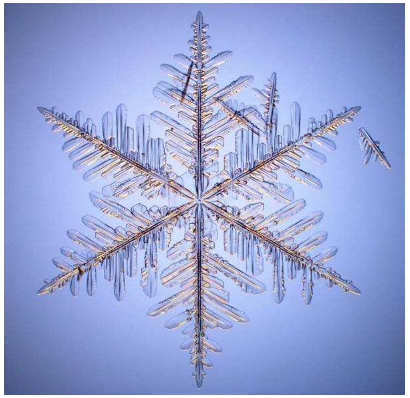
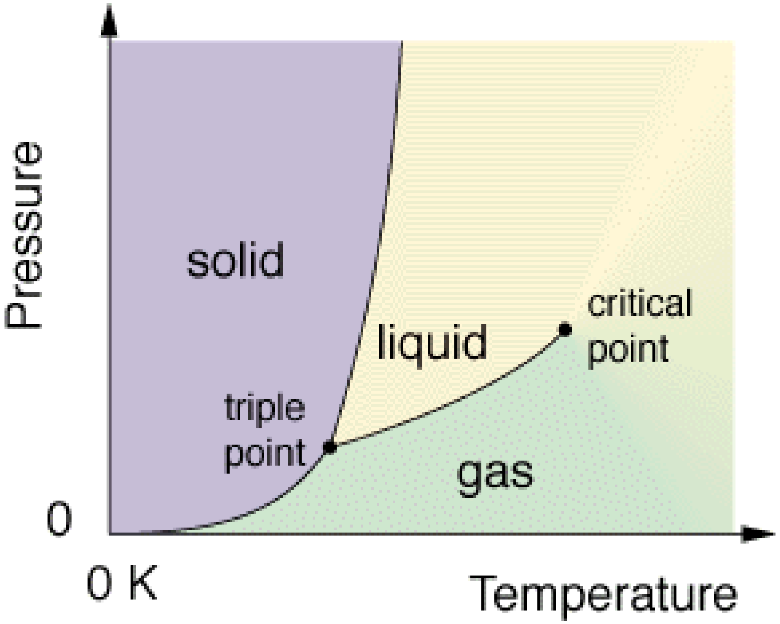
High Density QCD

T. Hatsuda
(Univ. Tokyo)



H_2O



Outline of this talk

1. Phase structure of QCD
   – what we know and what we do not know

2. Phases in hot QCD

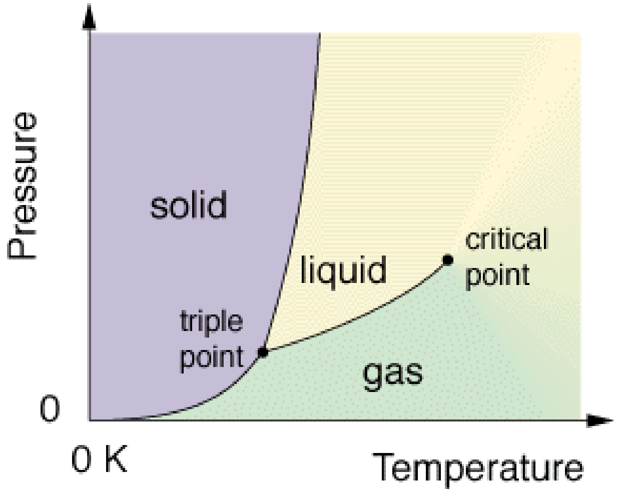
3. Phases in dense QCD

4. Future

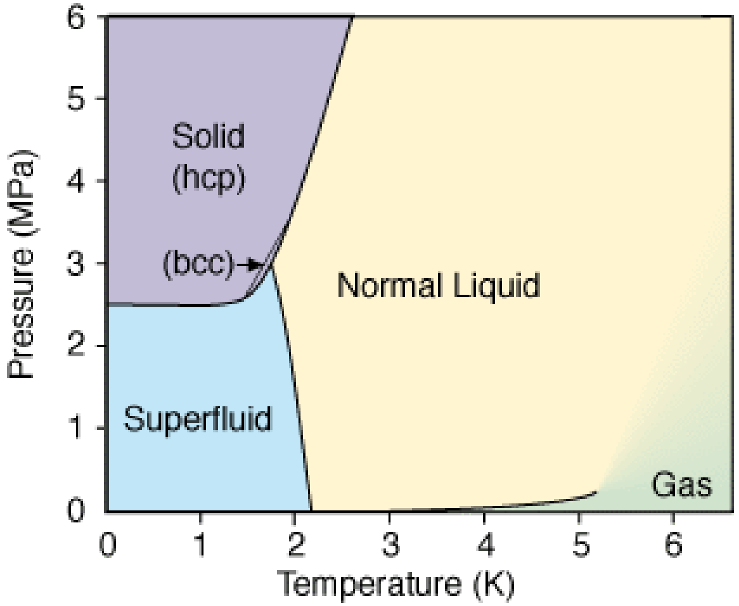
5. Summary

Quantum phases of ^{3}He & ^{4}He

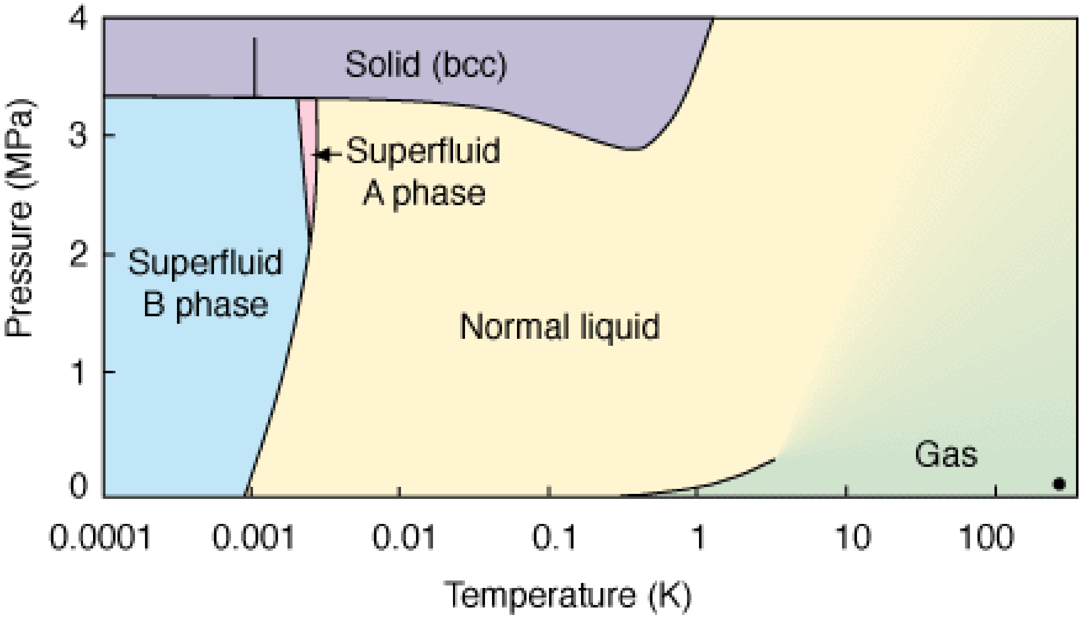
H_2O



^{4}He

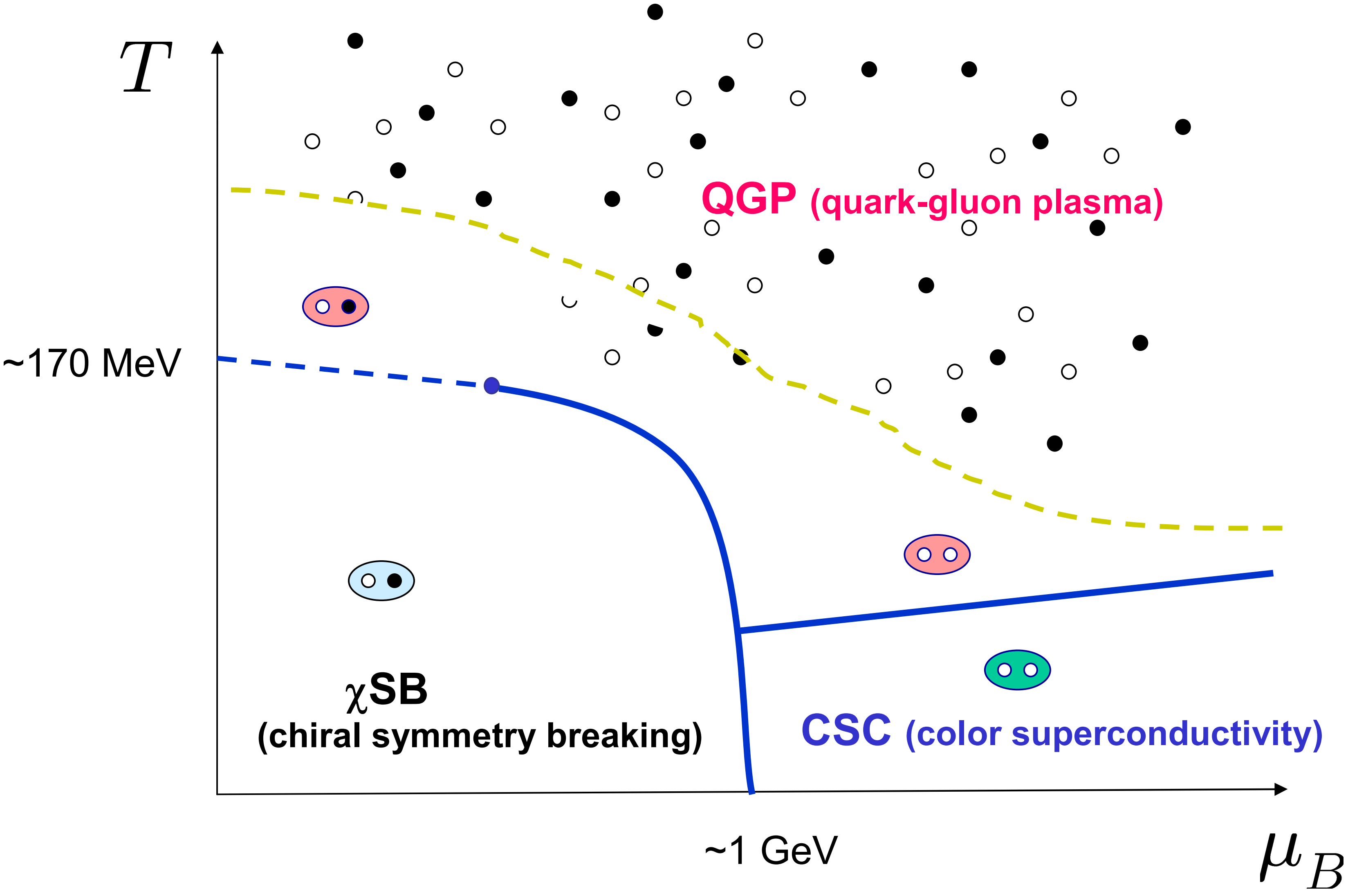


^{3}He



http://boojum.hut.fi/research/theory/typicalpt.html

Phases in QCD ? – a schematic picture --



Theory behind: Quantum Chromo Dynamics

1965 $SU_c(3)$ YM theory as a model of strong interaction

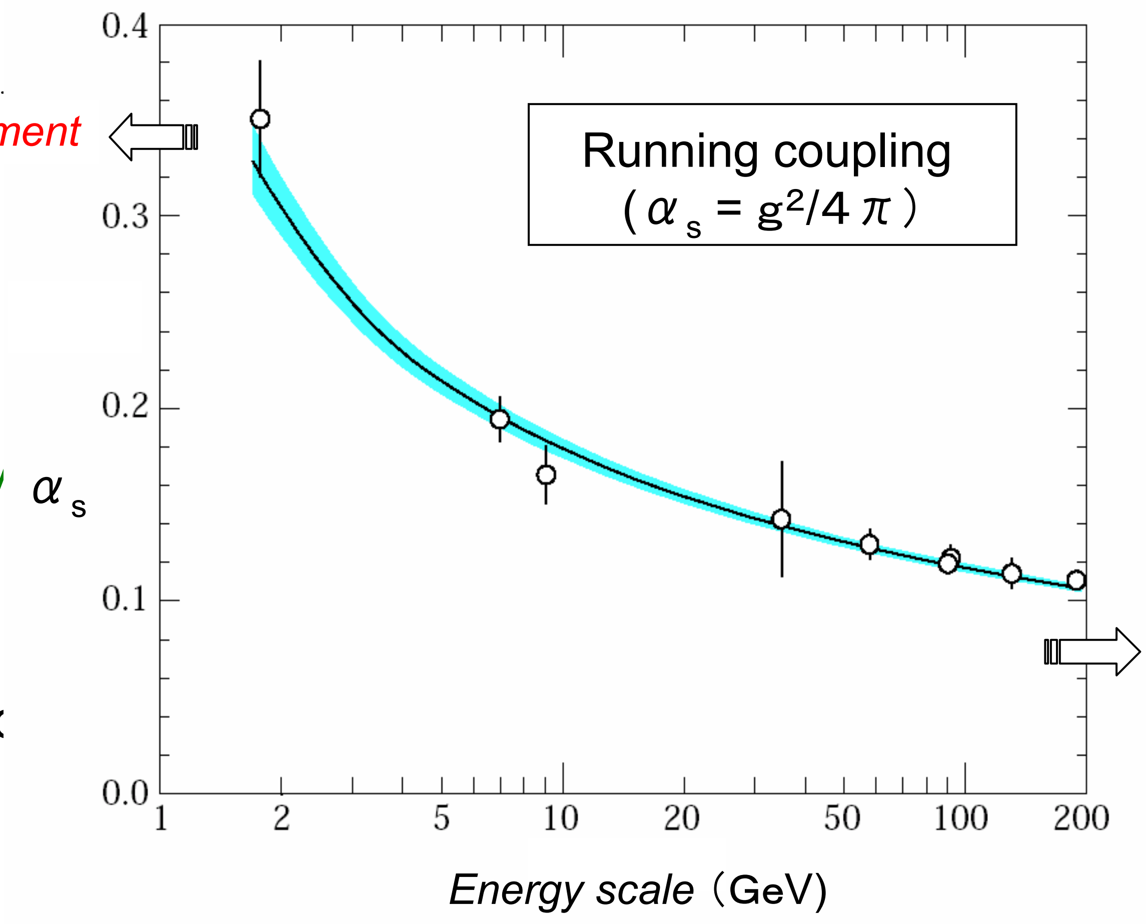
Color Confinement

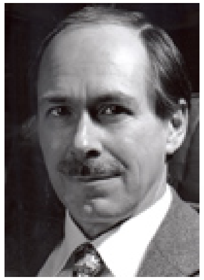
$\mathcal{L} = -$

1965-1972

Vany

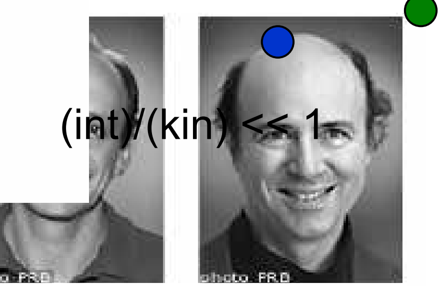
1973 Disc

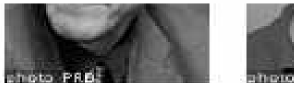




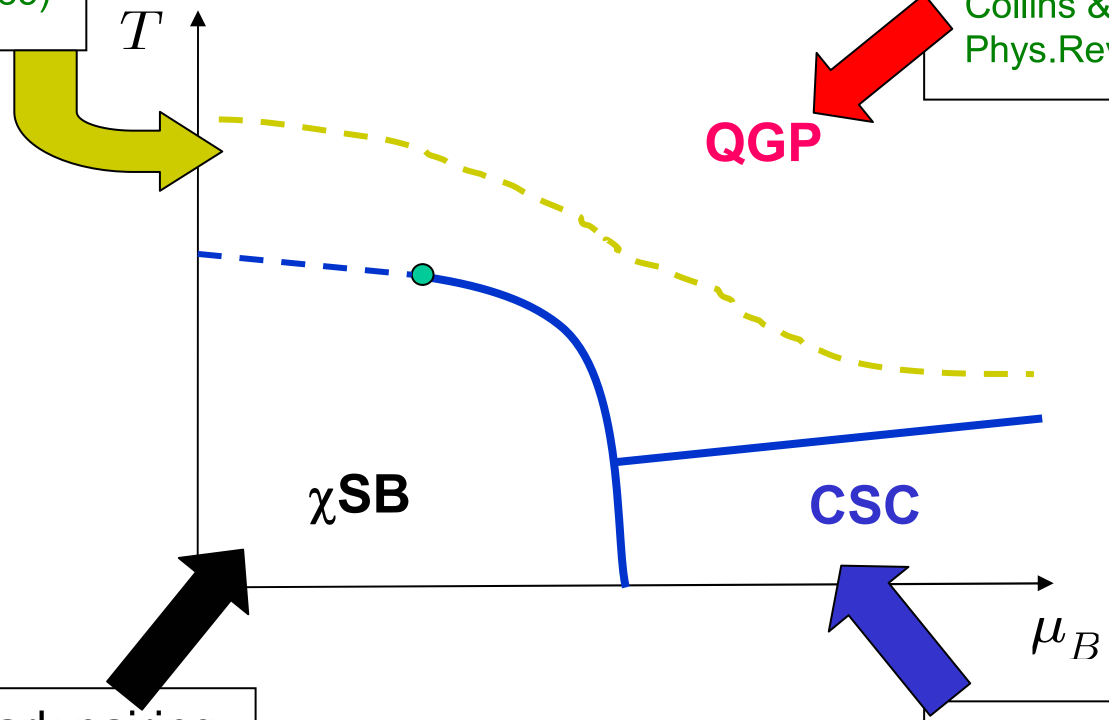
Asymptotic freedom

(int)/(kin) << 1





Origin of each “phase”



strong residual force
→ **pre-formed pairs**

Hatsuda & Kunihiro,
Phys.Rev.Lett. 55 (1985)

Asymptotic freedom
+ Debye screening
→ **deconfinement**

Collins & Perry,
Phys.Rev.Lett. 34 (1975)

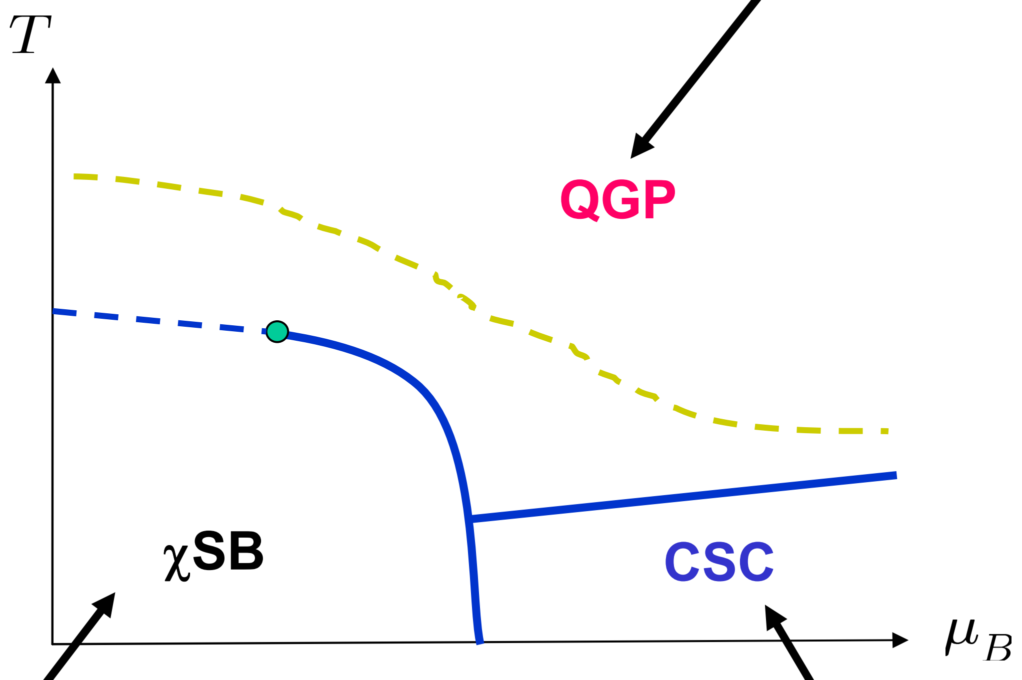
quark - anti-quark pairing
→ **chiral instability**

Nambu & Jona-Lasinio,
Phys.Rev. 122 (1961)

quark-quark pairing
→ **Cooper instability**

Bailin & Love,
Phys.Rep.107 (1984)

Symmetry of each “phase” (case for small m_{ud} with $m_s=\infty$)



$SU_C(3)\times[SU_L(2)\times SU_R(2)]\times U_B(1)$

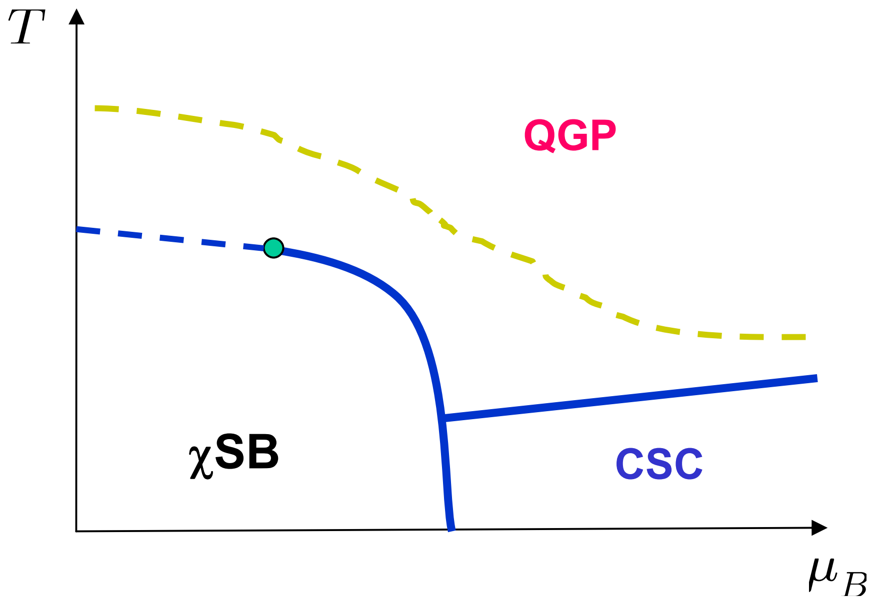
$SU_C(3)\times SU_{L+R}(2)\times U_B(1)$

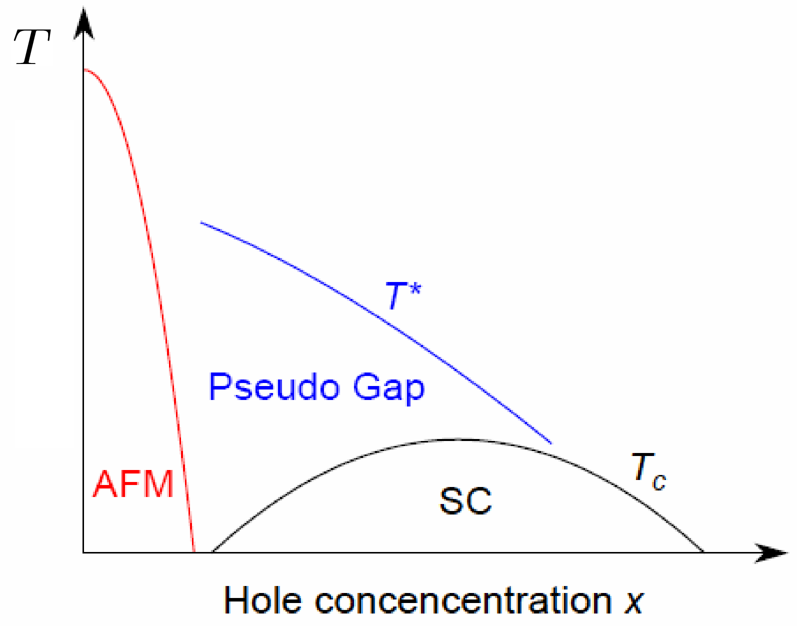
$\langle \bar{q}q\rangle \neq 0$

$SU_C(2)\times[SU_L(2)\times SU_R(2)]\times \widetilde{U}_{B+C}(1)$

$\langle qq \rangle \neq 0$

Similarity with high T_c superconductors

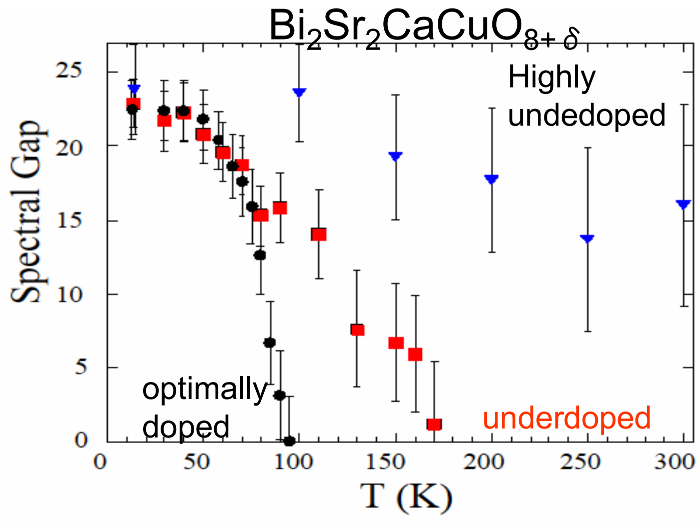




1. insulator – superconductor transition
2. strong coupling → pre-formed pairs?
3. similar physics in "cold-atoms" too

HTS – BEC – QCD connection ?

- Babaev, PRD ('00)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Kitazawa, Koide, Kunihiro & Nemoto, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)

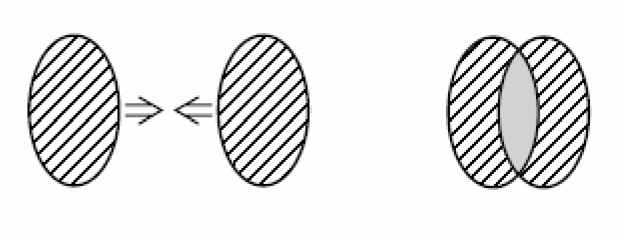


Experimental/Observational probes

Heavy ion collisions

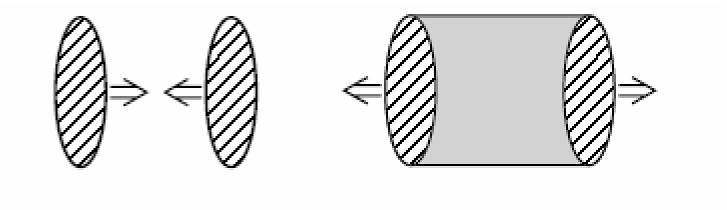
- low energies : (1-20) GeV/A, SIS (GSI), AGS (BNL), SPS (CERN)
  SIS100/300 (GSI), J-PARC (Japan)

  → dense matter

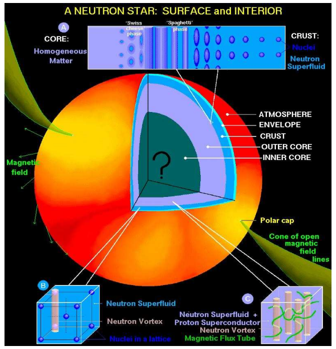


- high energies : (200-) GeV/A, RHIC (BNL), LHC (CERN)

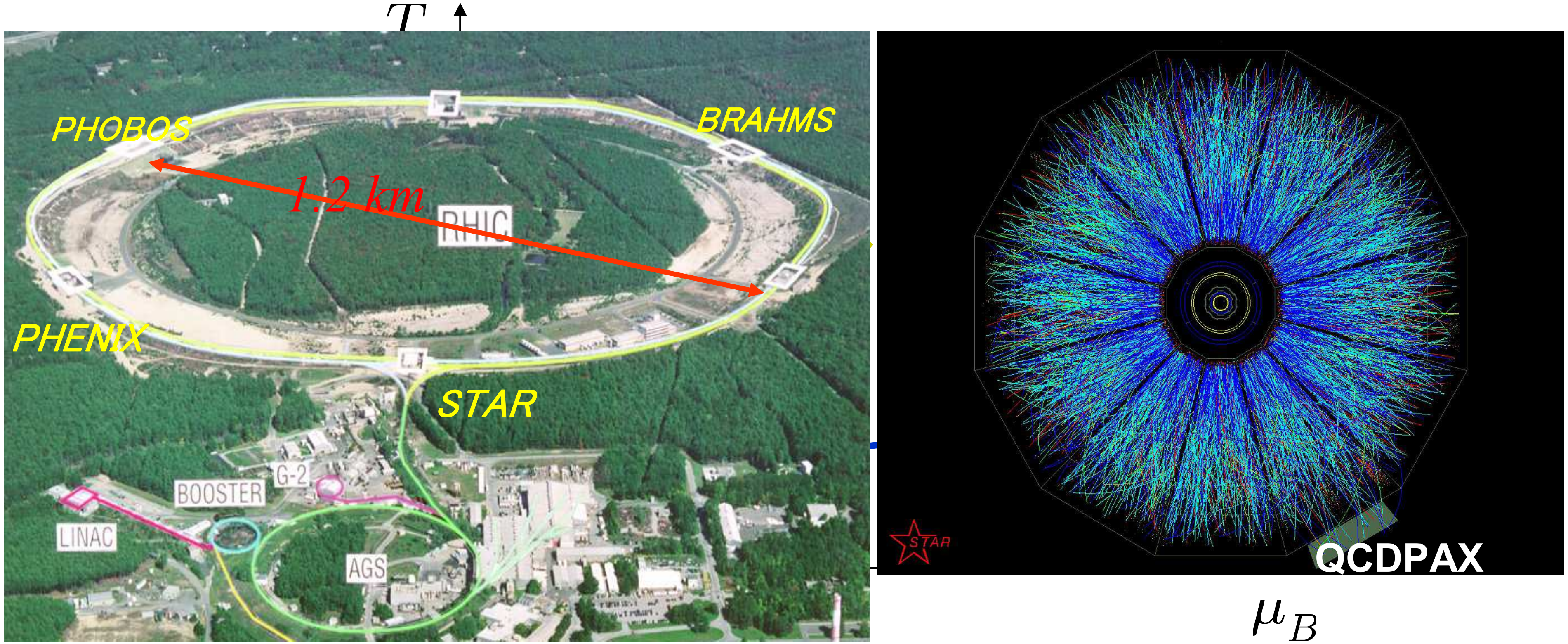
  → hot matter



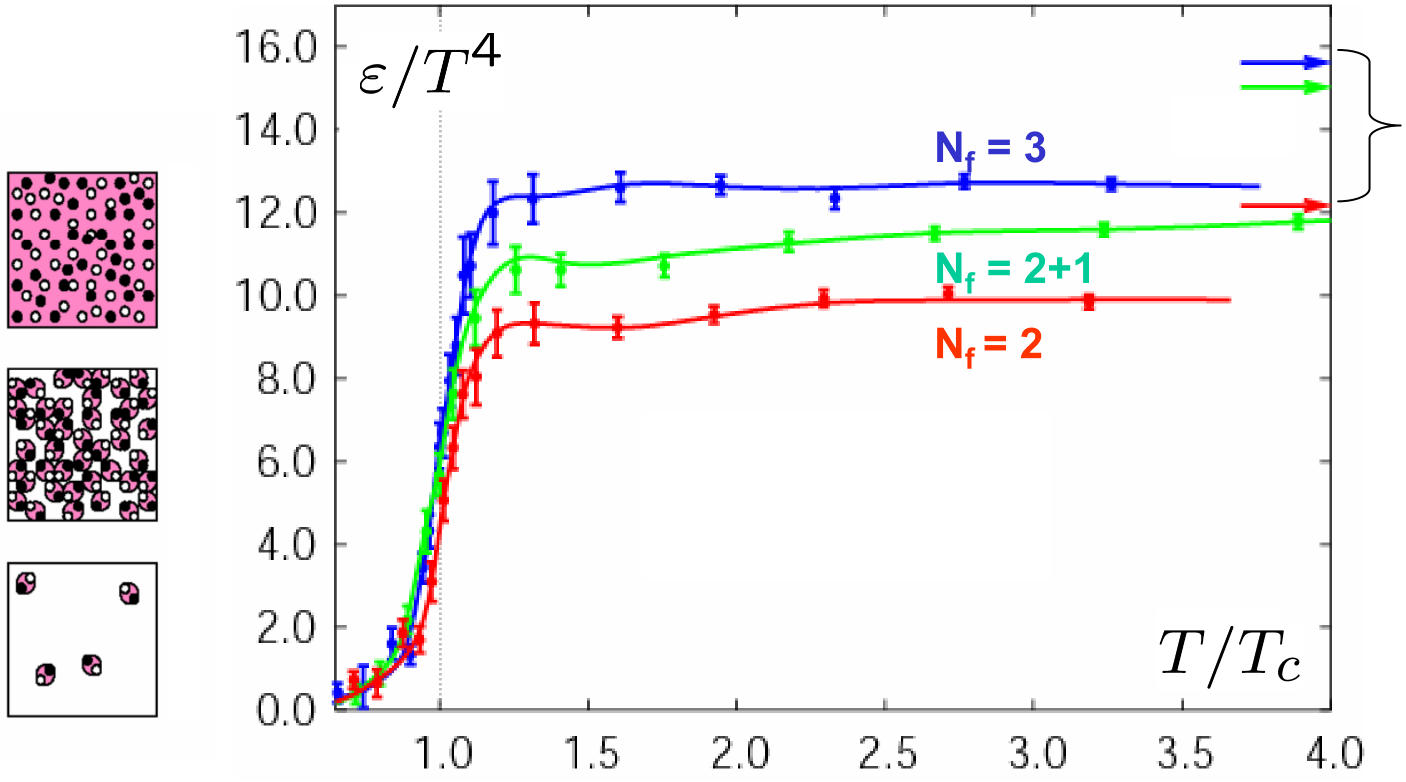
Compact stars : dense matter



Phases in hot QCD



Equation of state (EoS) (lattice QCD simulation)



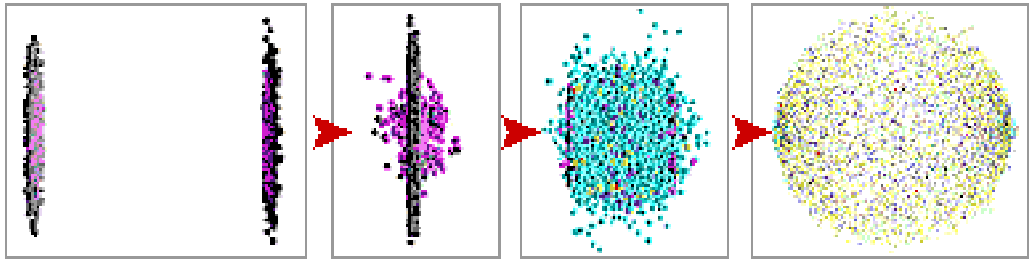
Ideal gas limit

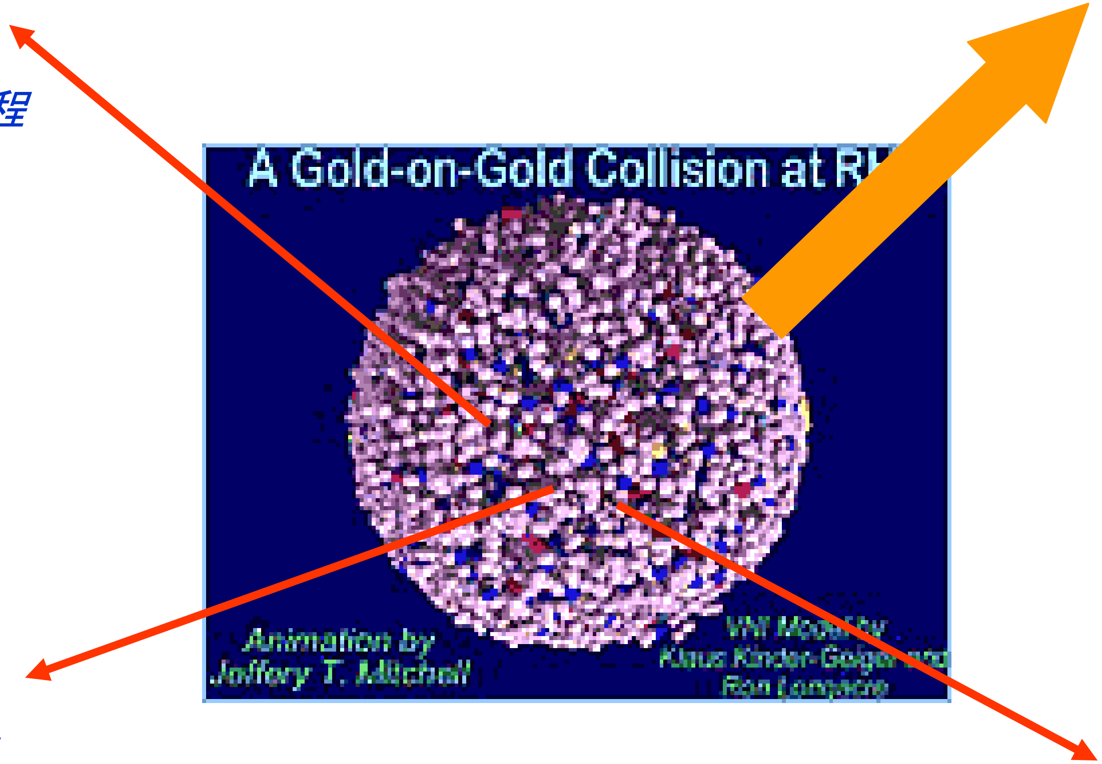
Karsch, Lect.Notes Phys. 583 ('02) 209

SPS

RHIC

LHC





γ線

内部温度
プラズマ反応過程

ハドロン放出

熱・化学平衡
エネルギー密度
流体発展
火の玉サイズ

レプトン対

カイラル対称性
色デバイ遮蔽

ジェット放出

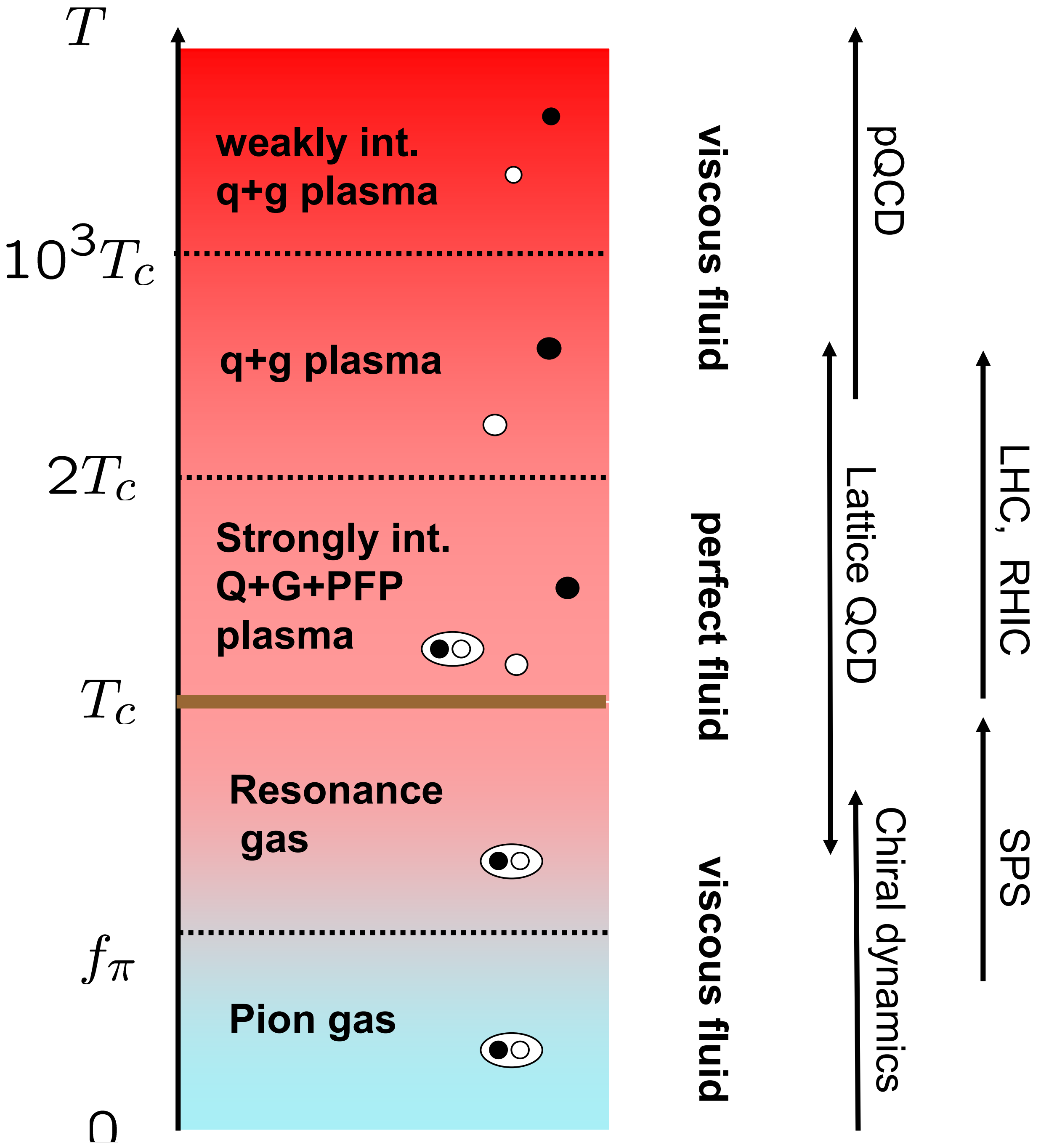
プラズマ密度

時空発展

e.g. Relativistic hydrodynamics
-- full (3+1)-dim. Simulations --
Hirano, PRL86 ('01), Hirano & Nara, PRL91 ('03)

A modern “picture” of hot QCD from lattice QCD

- Asakawa & Hatsuda, Phys. Rev. Lett. (’04)
- Nakamura & Sakai, Phys. Rev. Lett. (’05)



T

10^3T_c

$2T_c$

T_c

f_π

0

weakly int. q+g plasma

q+g plasma

Strongly int. Q+G+PFP plasma

Resonance gas

Pion gas

viscous fluid

perfect fluid

viscous fluid

pQCD

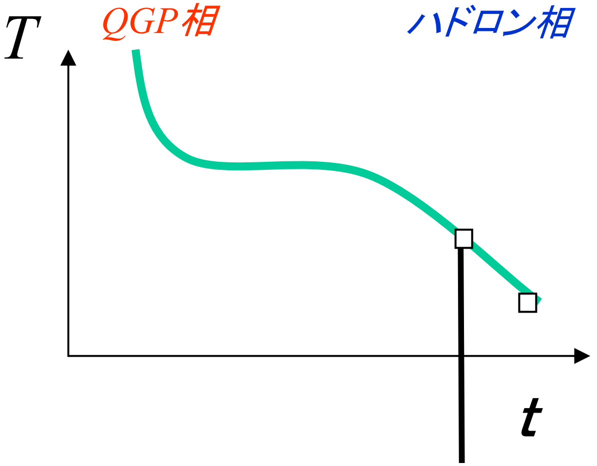
Lattice QCD

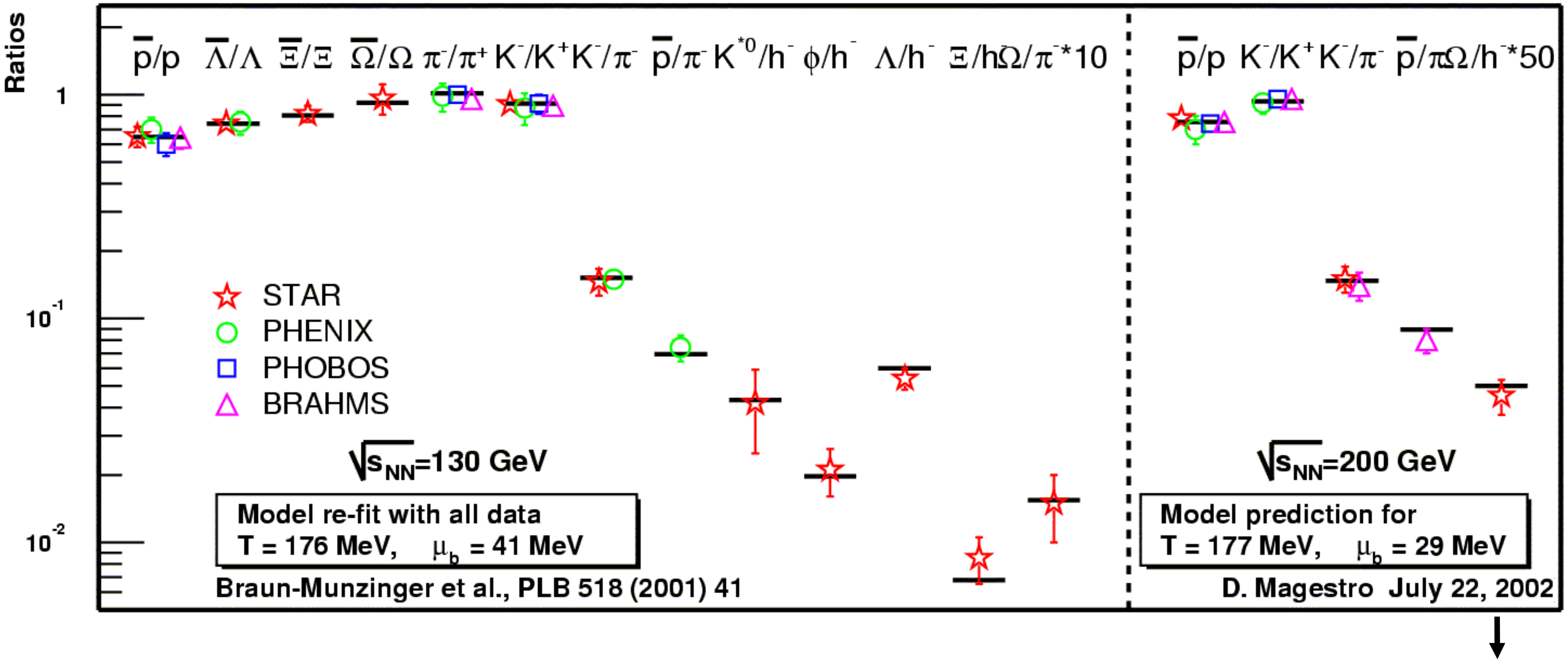
Chiral dynamics

LHC, RHIC

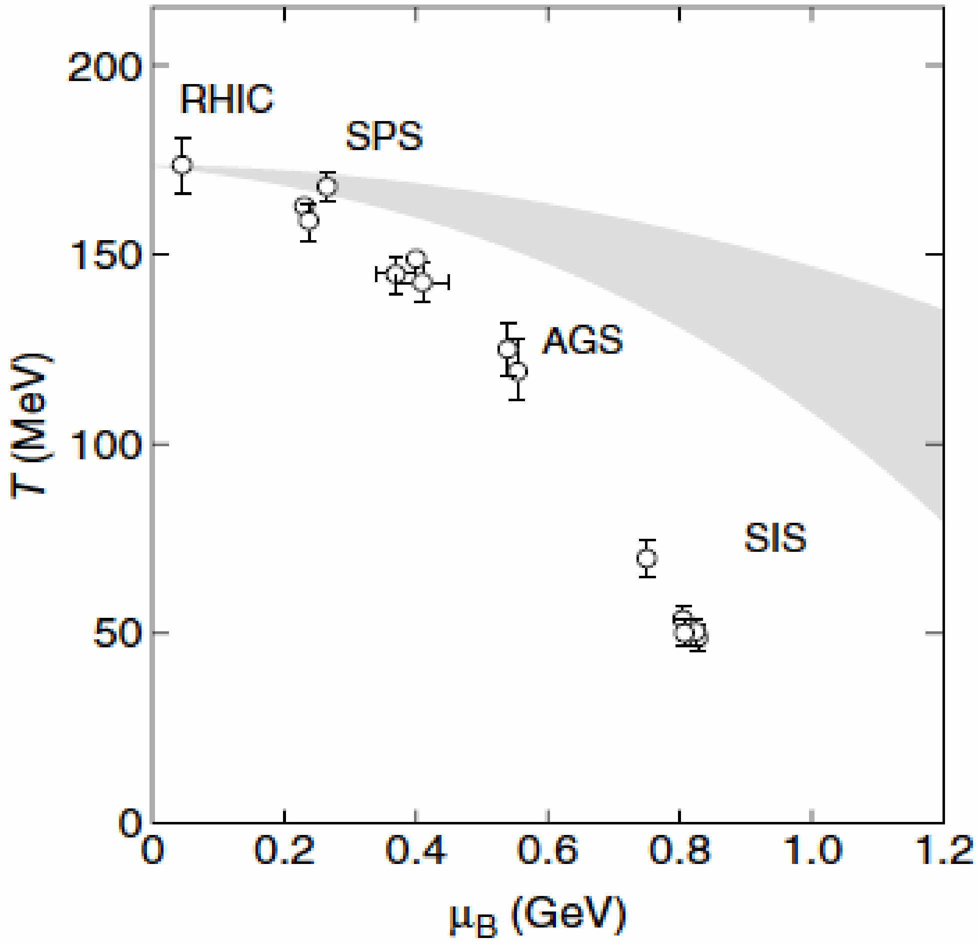
SPS

化学平衡 @ RHIC

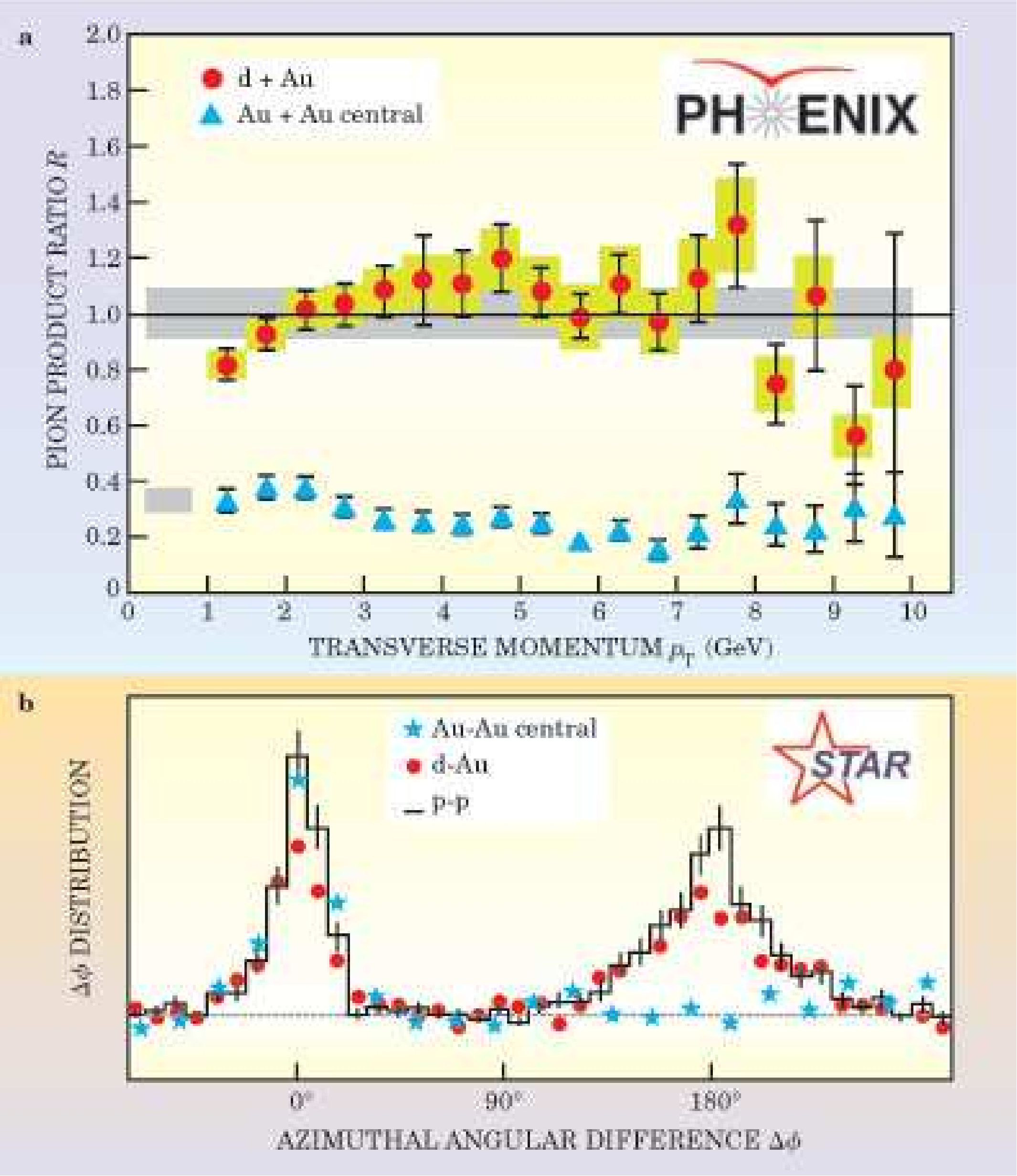


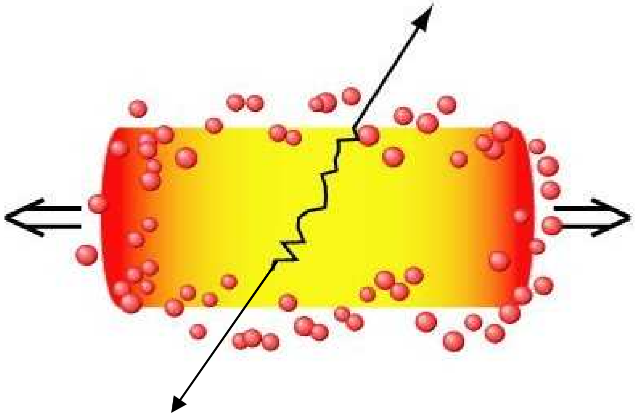


化学凍結 $T = 177$ MeV, $\mu_B = 29$ MeV


RHIC
SPS
AGS
SIS
200
150
100
50
0
T (MeV)
0
0.2
0.4
0.6
0.8
1.0
1.2
μ_B (GeV)

ジェット抑制の発見@RHIC

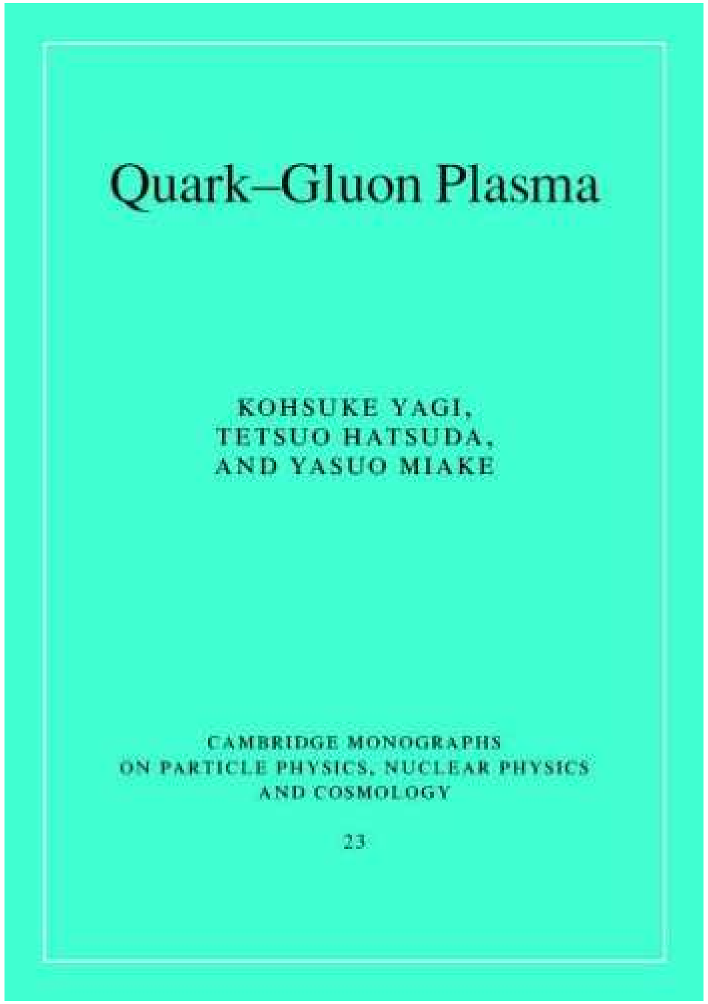




Bjorken’s formula

$$\varepsilon_{\rm Bj} = \frac{1}{\pi R^2 \tau_0} \frac{dE_{\rm T}}{dy}\bigg|_{y\simeq 0}$$

$\varepsilon_{\rm Bj} >$ 5 GeV/fm^3
~ 5 $\varepsilon_{\rm crit}$
~ 30 ε_0



Quark–Gluon Plasma

KOHSUKE YAGI,
TETSUO HATSUDA,
AND YASUO MIAKE

CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY

23

published, Dec. 15, 2005

1. What is quark-gluon plasma

Part I. Basic Concept of Quark-Gluon Plasma:

2. Introduction to QCD
3. Physics of quark-hadron phase transition
4. Field theory at finite temperature
5. Lattice gauge approach to QCD phase transitions
6. Chiral phase transition
7. Hadronic states in hot environment

Part II. QGP in Astrophysics:

8. QGP in the early universe
9. Compact stars

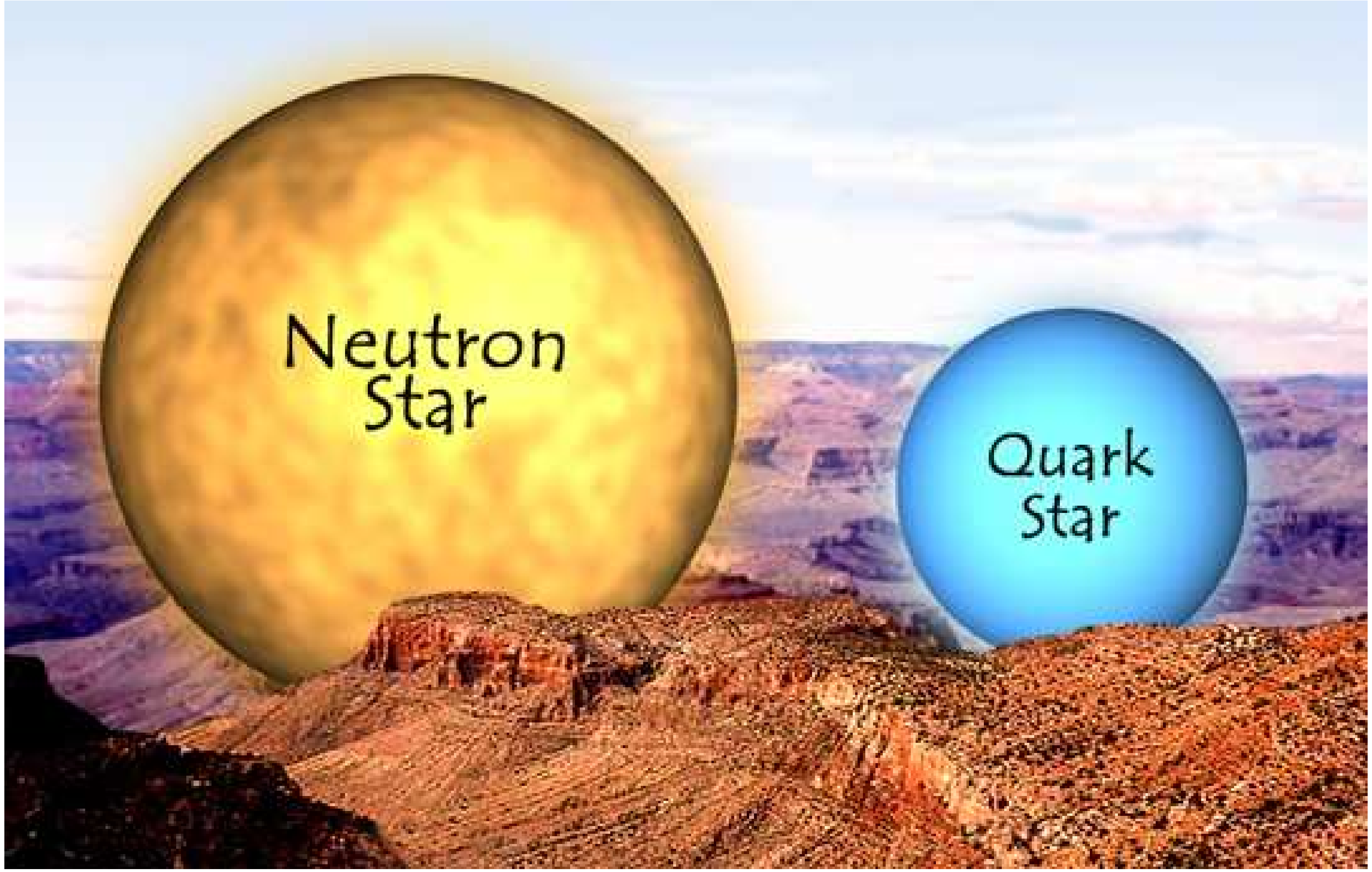
Part III. QGP in Relativistic Heavy Ion Collisions:

10. Introduction to relativistic heavy ion collisions
11. Relativistic hydrodynamics for heavy ion collisions
12. Transport theory for pre-equilibrium process
13. Formation and evolution of QGP
14. Fundamentals of QGP diagnostics
15. Results from CERN-SPS experiments
16. First results from BNL-RHIC
17. Detectors in relativistic heavy ion experiments

http://utkhii.px.tsukuba.ac.jp/cupbook/index.html

Appendices A-H:
120 Exercises

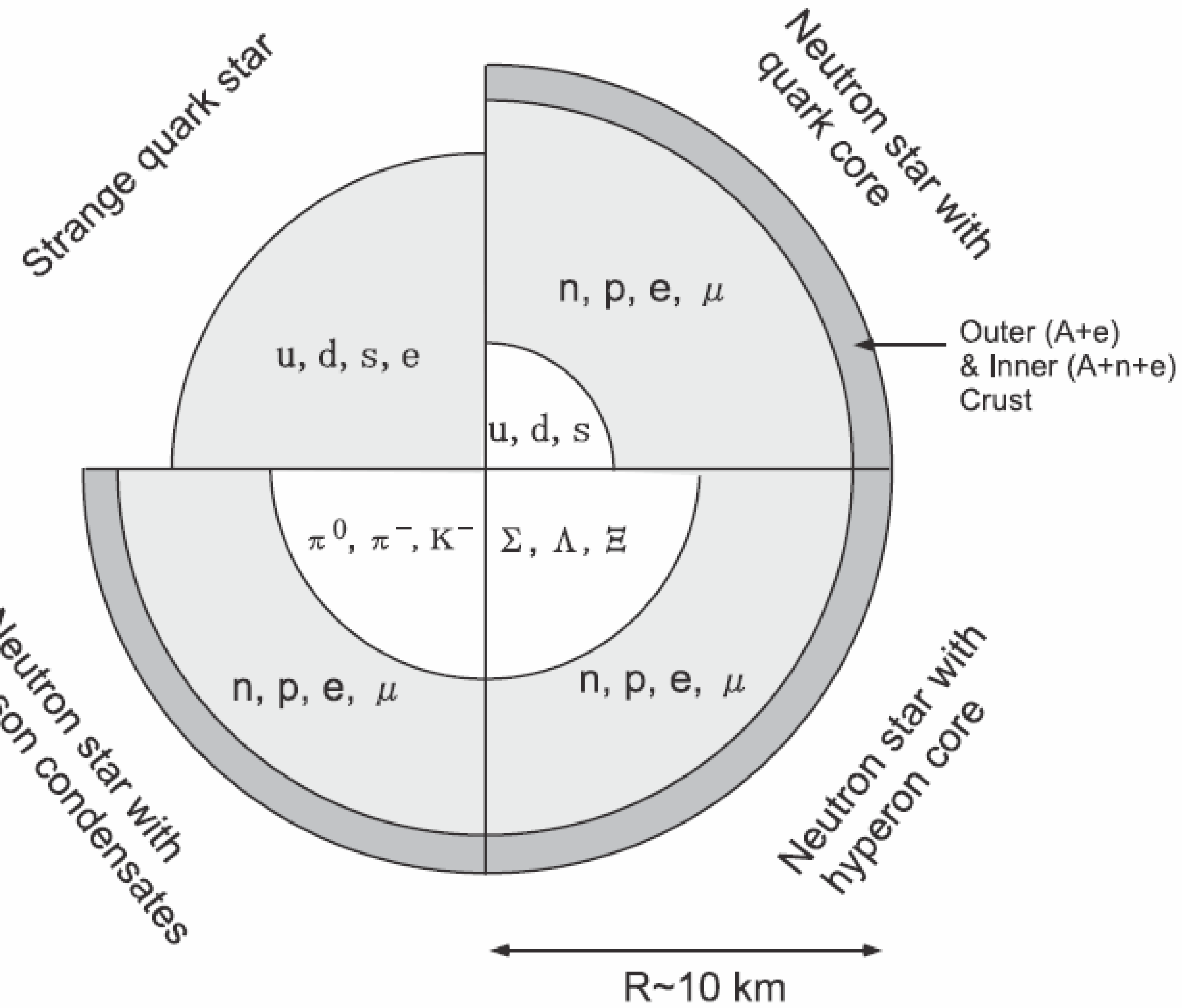
Phases in dense QCD

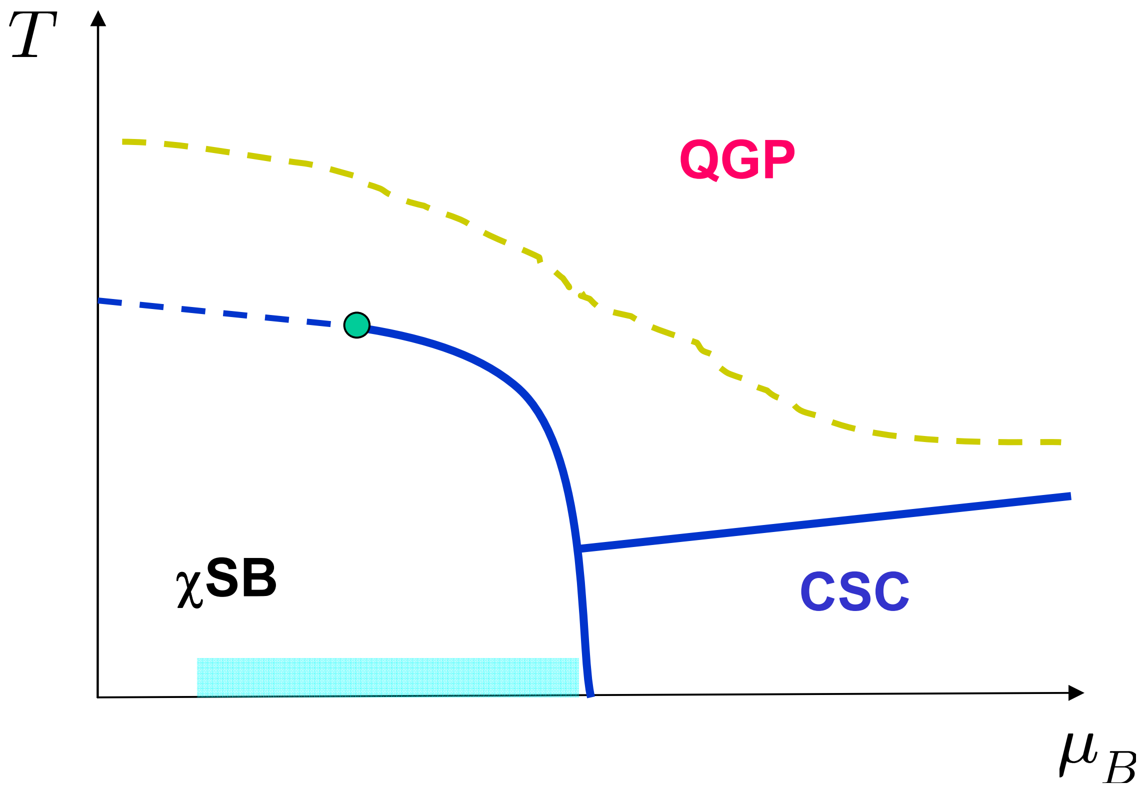


Baade-Zwicky ('34)   N. Itoh ('70), E. Witten ('84)

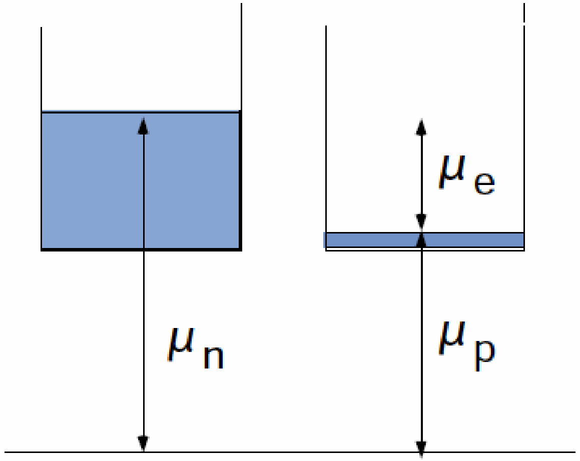
http://chandra.harvard.edu/resources/illustrations/neutronStars.html

Structure of compact stars




T
QGP
χSB
CSC
μ_B

Standard neutron star matter : n, p, e^- with $n \rightleftarrows p + e^-$

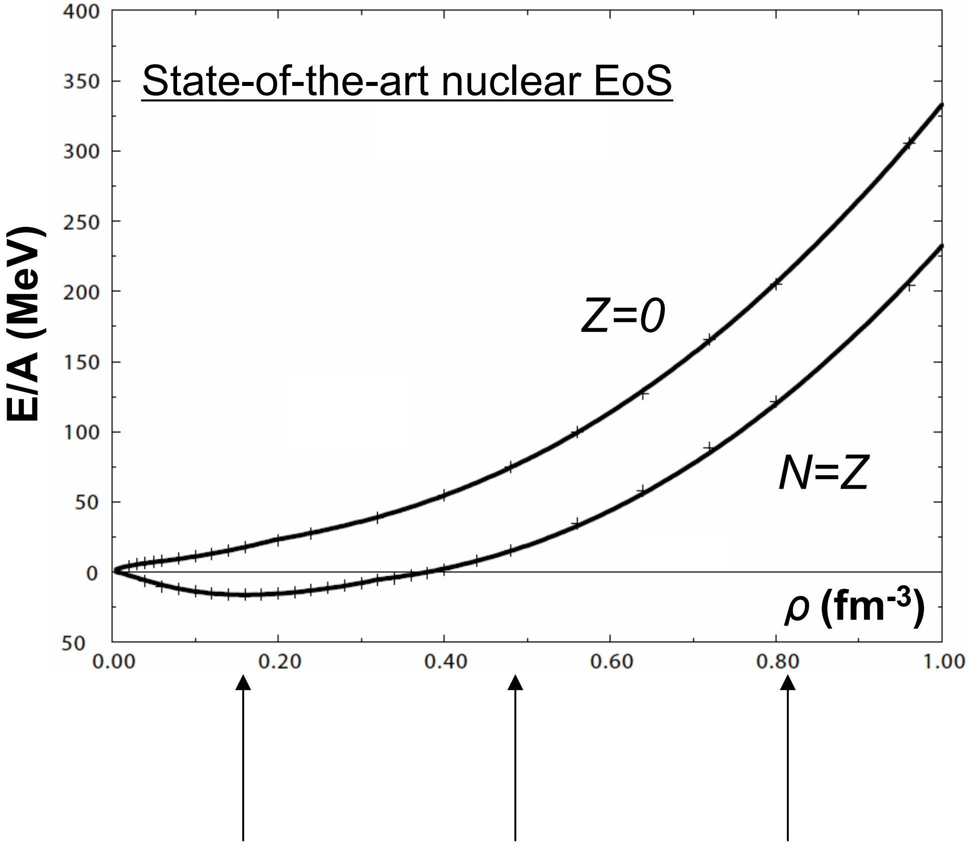


APR's EoS

- modern N-N int. (> 1994)

  (4301 N-N data fittied with χ^2/dof ~1)
- relativistic corrections
- N-N-N interactions
- **Variational method**

Akmal, **P**andharipande & **R**avenhall, PRC58 ('98)

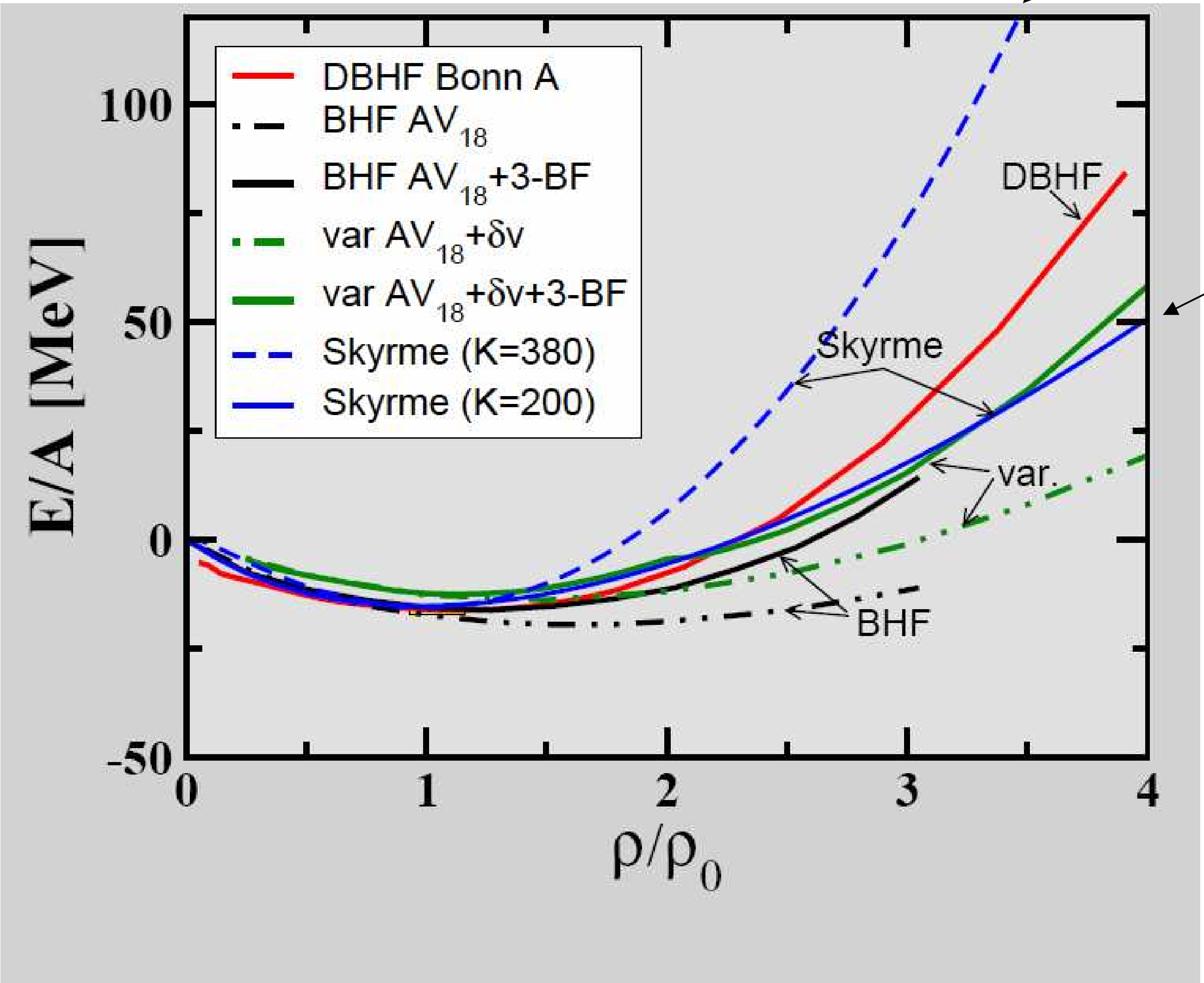


ρ_0 $= 0.16$ fm^{-3}

$3\rho_0$

$5\rho_0$

EoS of nuclear matter (N=Z) from ab initio calculations



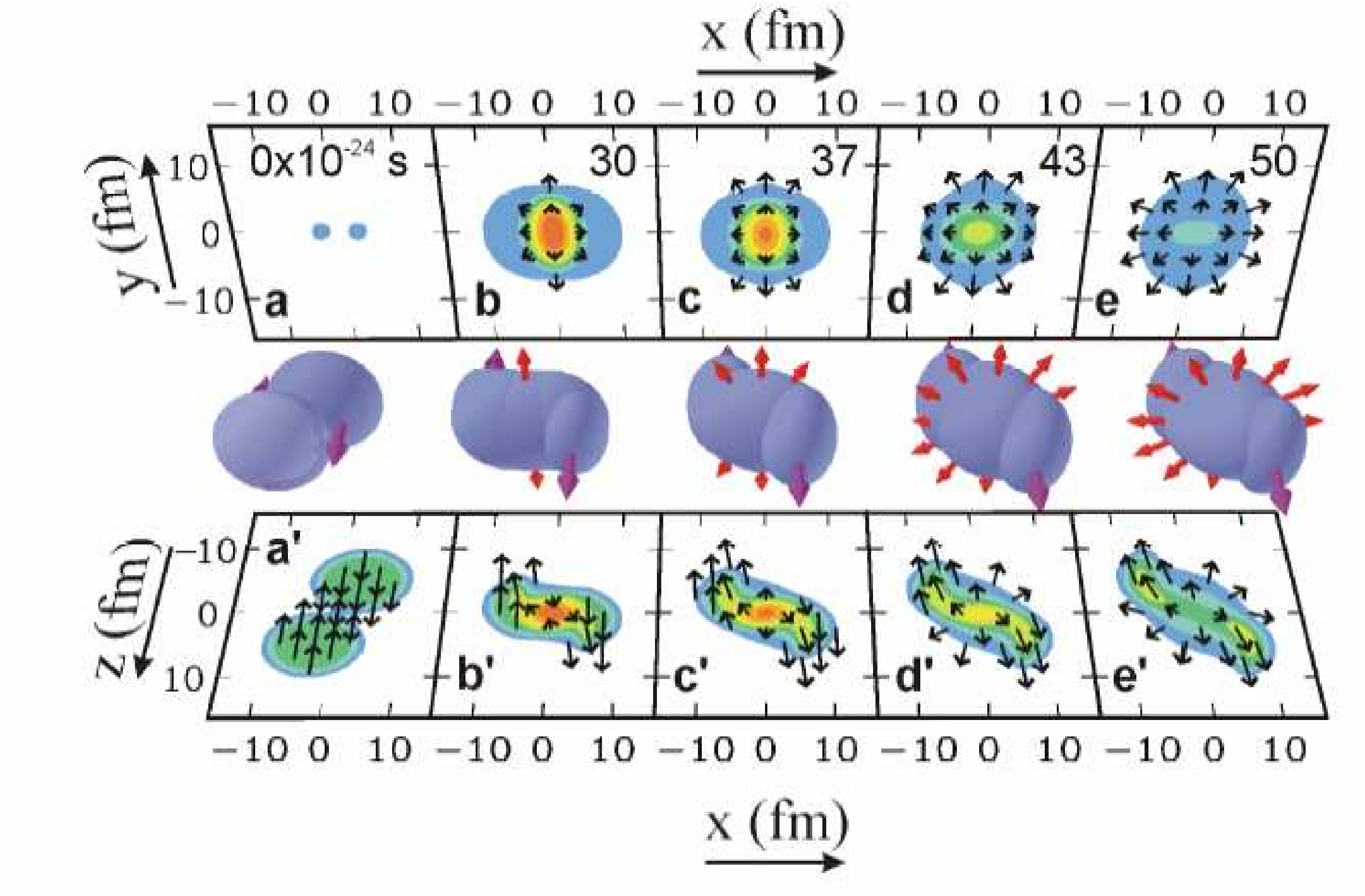
Hard EoS

Soft EoS

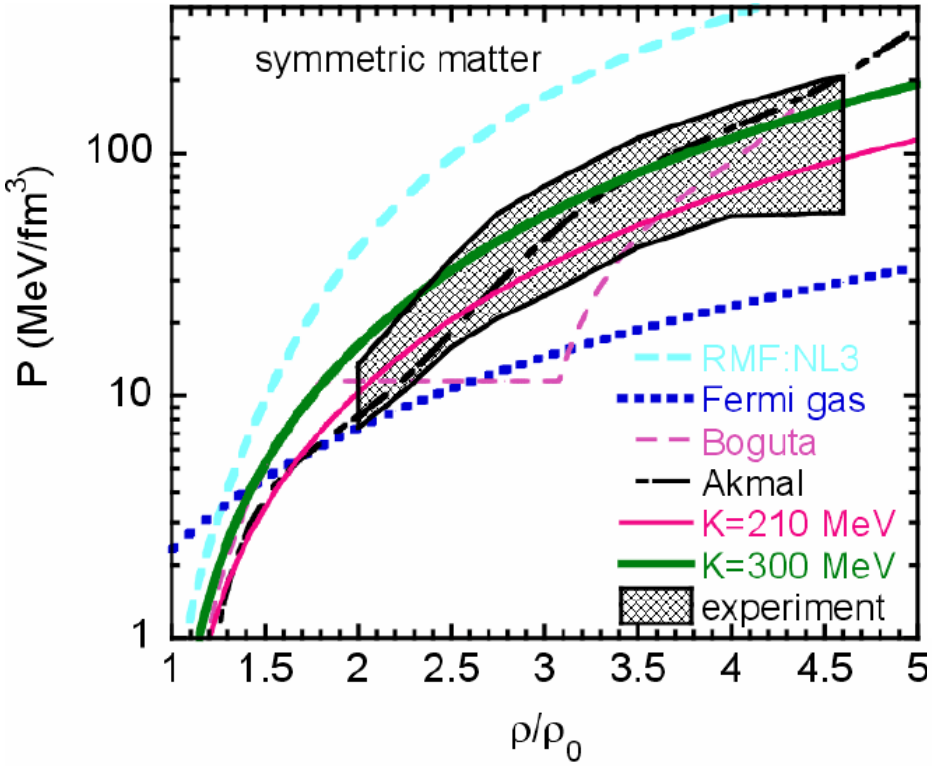
Fuchs and Walter, nucl-th/0511070

Collective flow in low-energy heavy ion collisions and EoS
semi-classical Boltzmann-Vlasov approach

$D\ f(x,p,t) = C[f]$ → $dN/d\phi = 1 + v_1 \cos\phi + v_2 \cos 2\phi + \ldots$

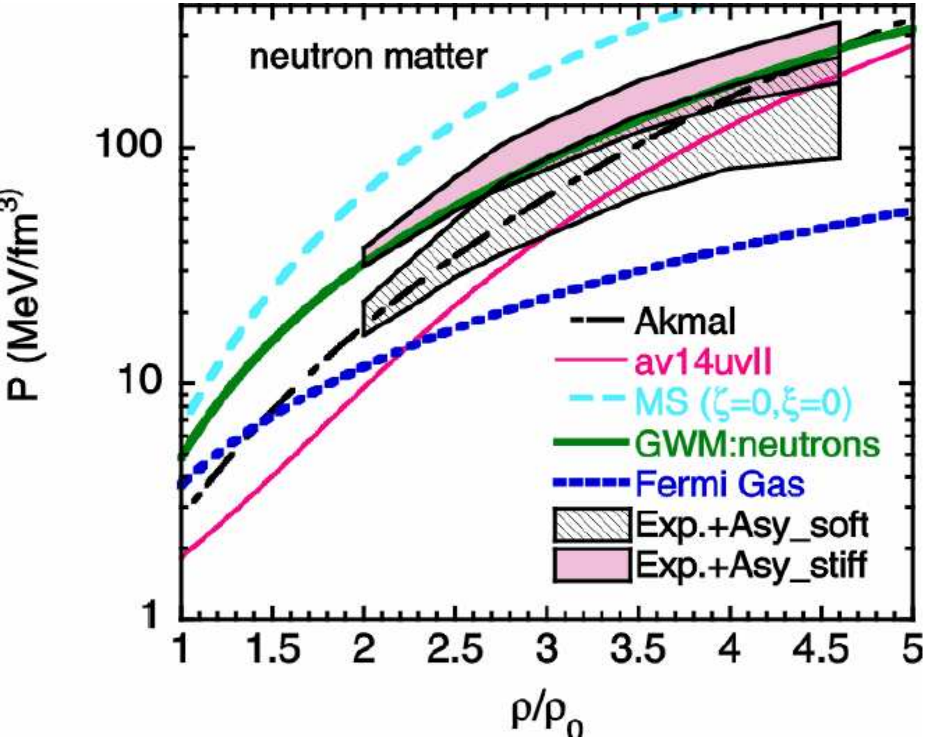


Au + Au, E_{lab} = 2GeV/A
Danielewicz et al, Science 298 (2002)



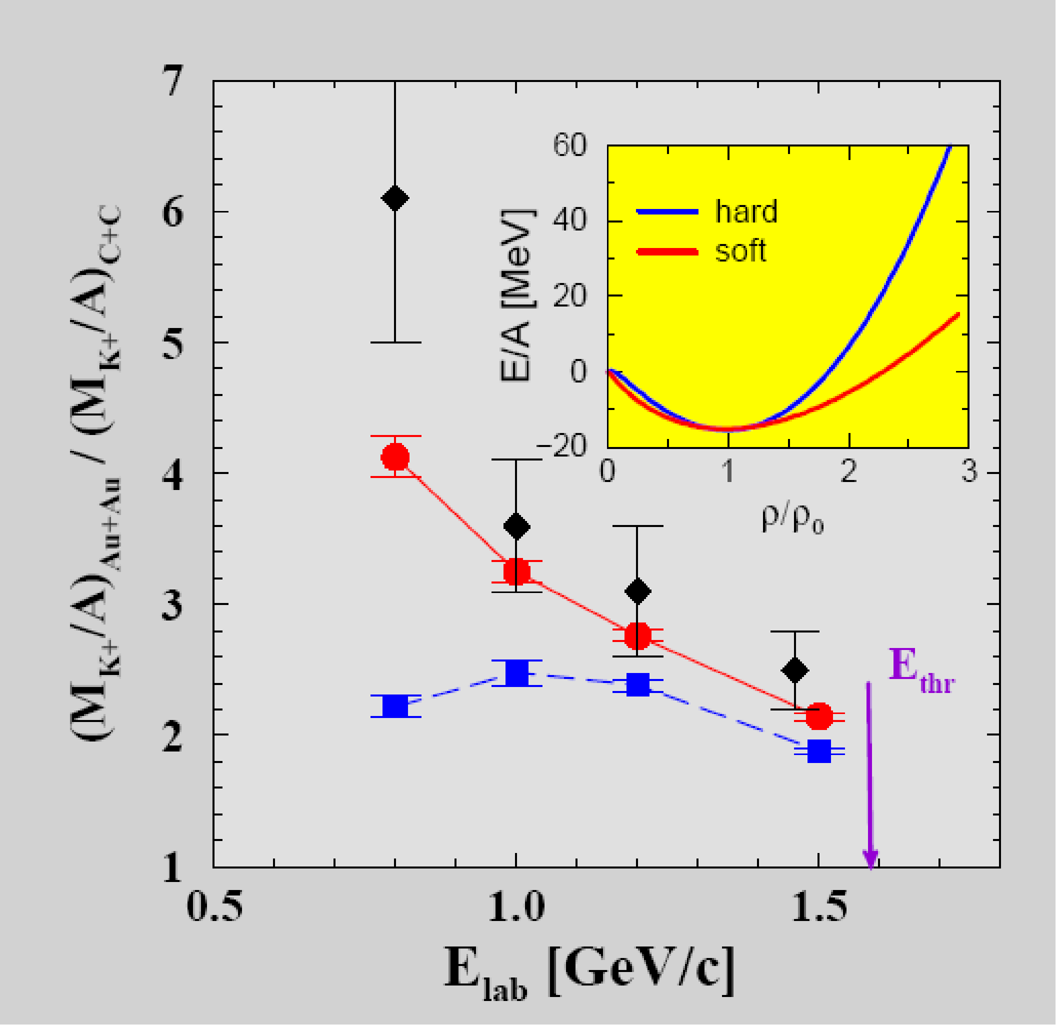
Constraint from Flow Data in SIS-AGS energies

Soft EOS is preferred



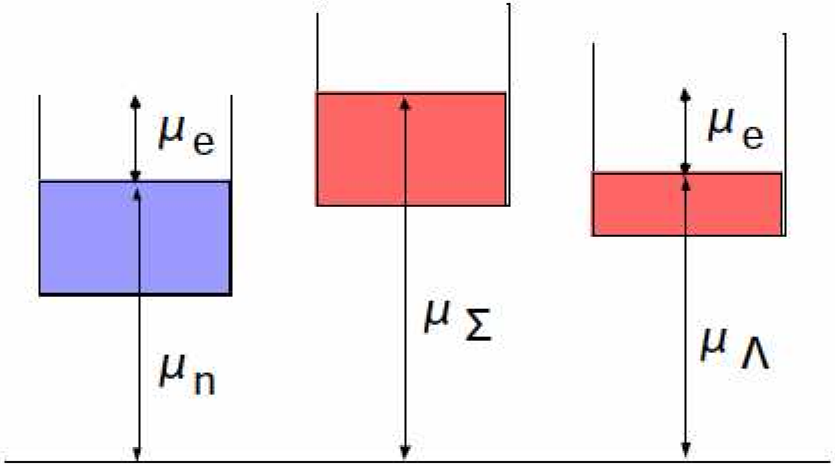
Danielewicz et al, Science 298 (2002)

Sub-threshold K^+ production at SIS ➡ **Soft EOS is preferred**



Fuchs and Walter, nucl-th/0511070

Hyperonic matter : n, p, Σ^-, Λ, e^- with $\Sigma^- \rightleftarrows n+e^-$, $\Lambda \rightleftarrows n$

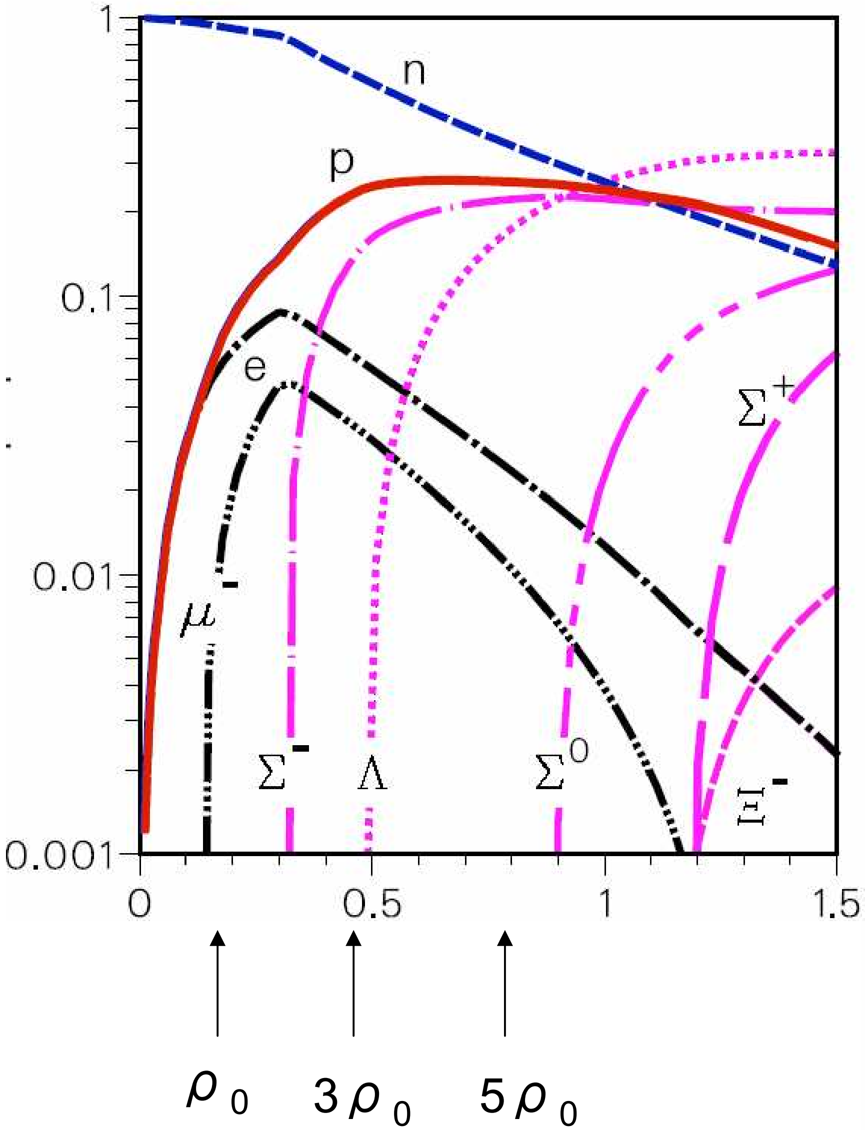


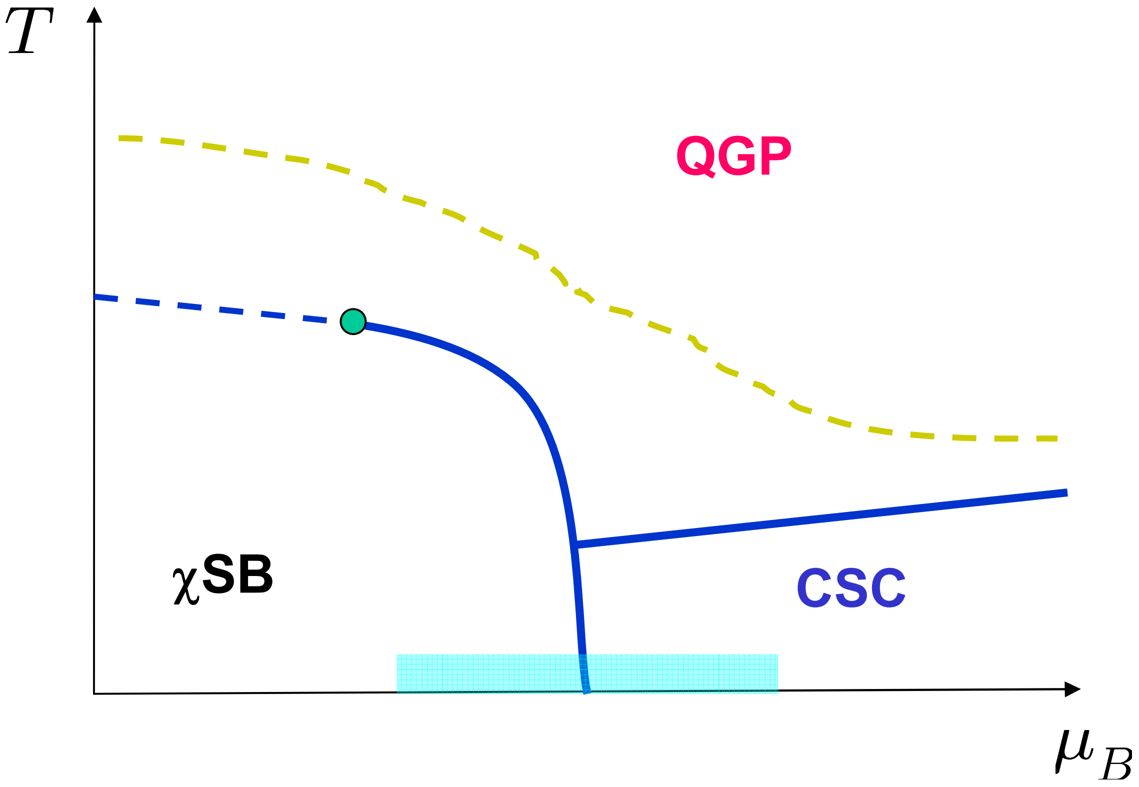
Remarks

- ● Hyperon mixing may start at (2-3) ρ_0
- ● Y-N int. not well understood

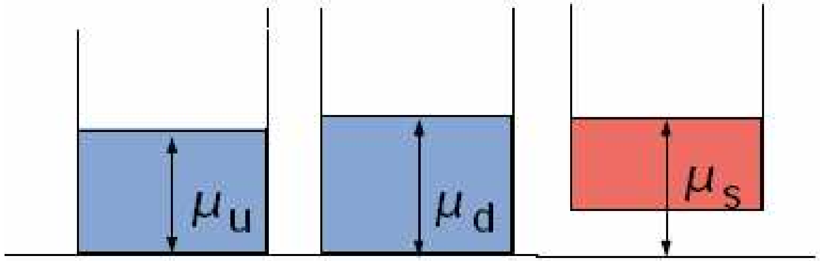
⟷ **J-PARC ('08)**

Glendnning, PRC64 ('01)



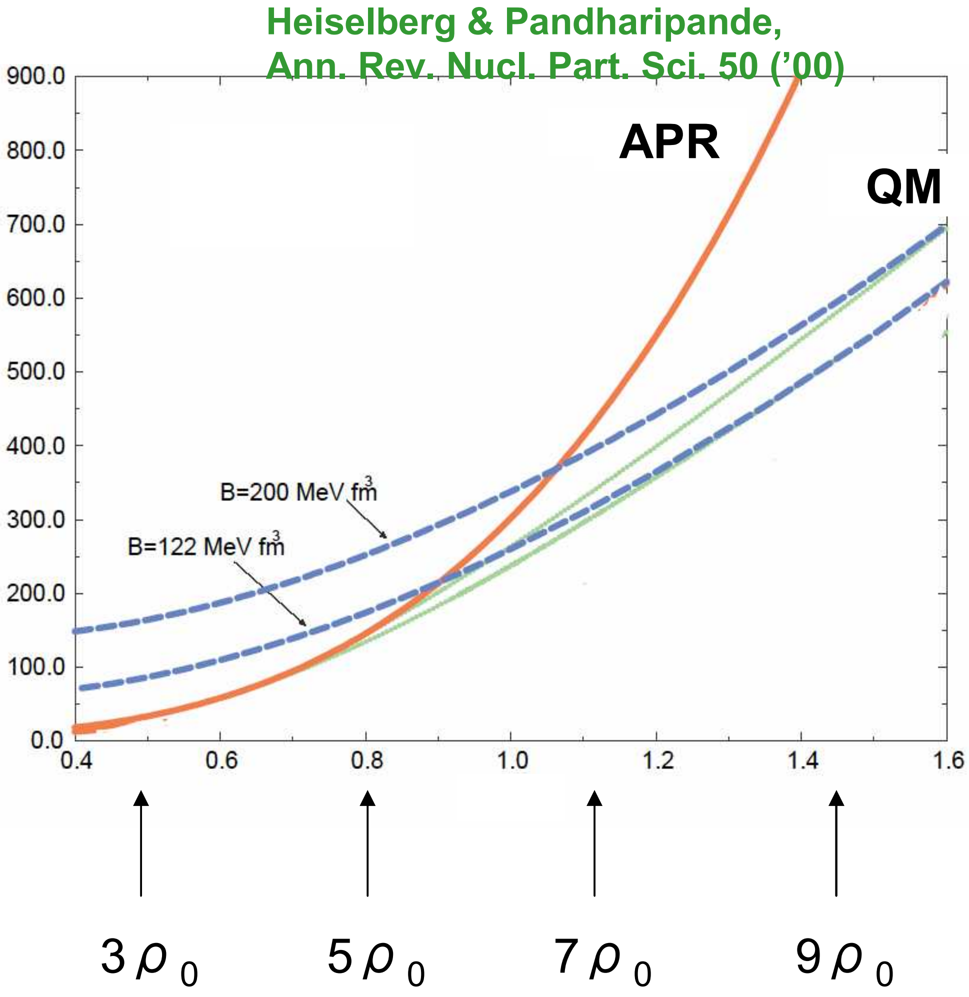

T
QGP
χSB
CSC
μ_B

Quark matter : u, d, s, e^- with $d \rightleftarrows u + e^-$, $d \rightleftarrows u + e^-$, $s \rightleftarrows d$

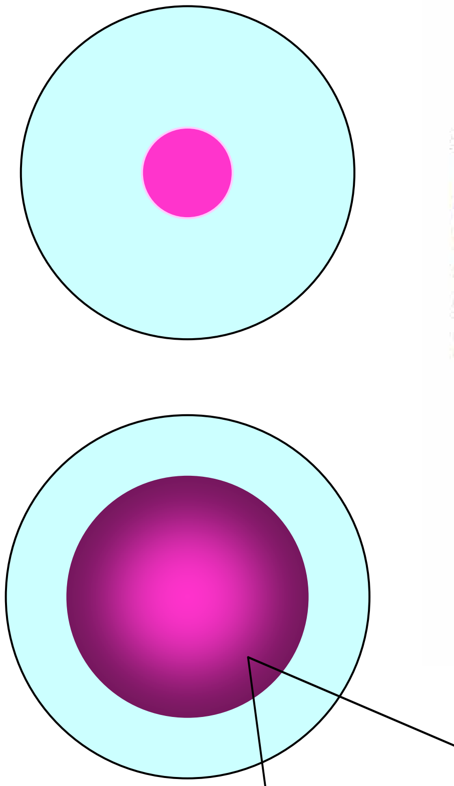


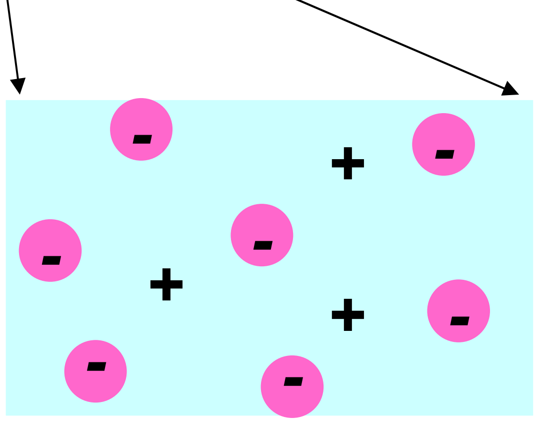
Remarks

- **Mixed phase may start even at ~ 3 ρ_0**
- sensitive to **quark matter EOS**

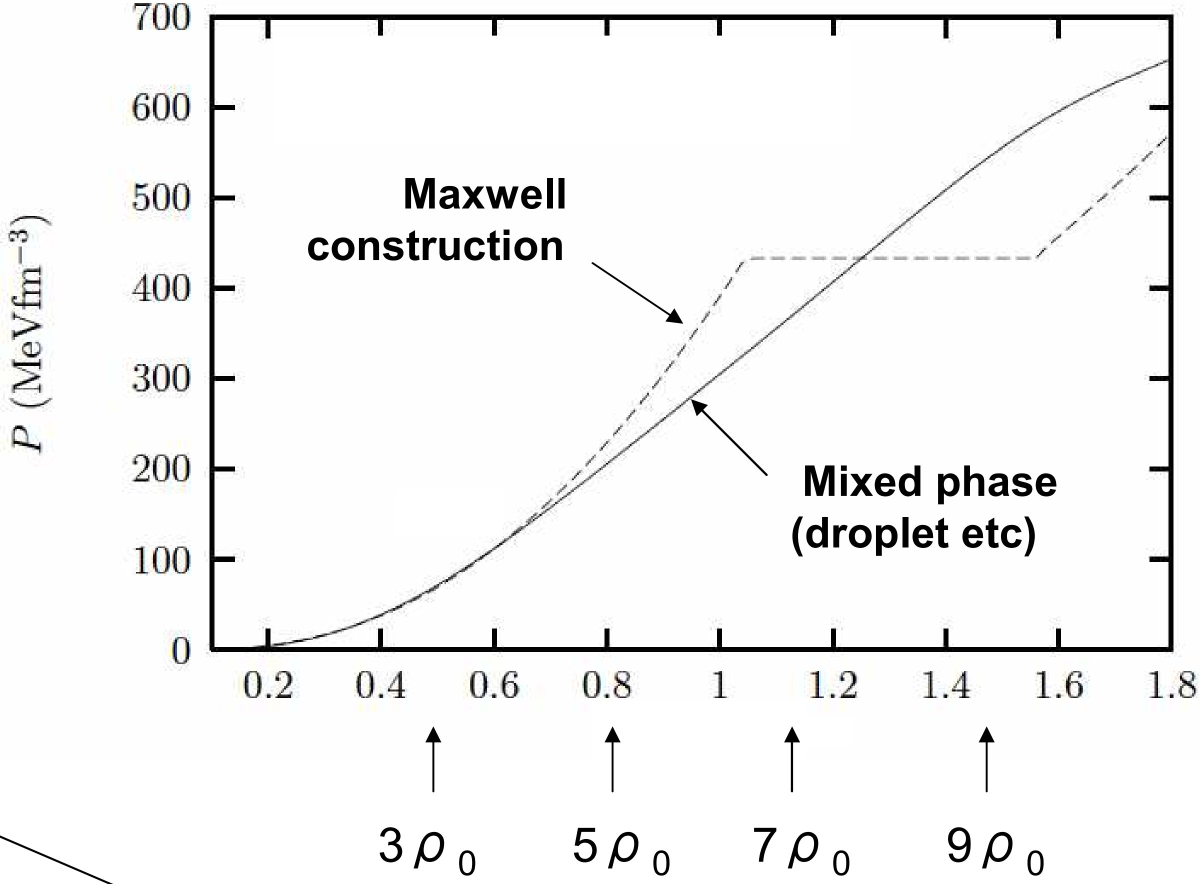

Heiselberg & Pandharipande, Ann. Rev. Nucl. Part. Sci. 50 ('00)

NM-QM mixture





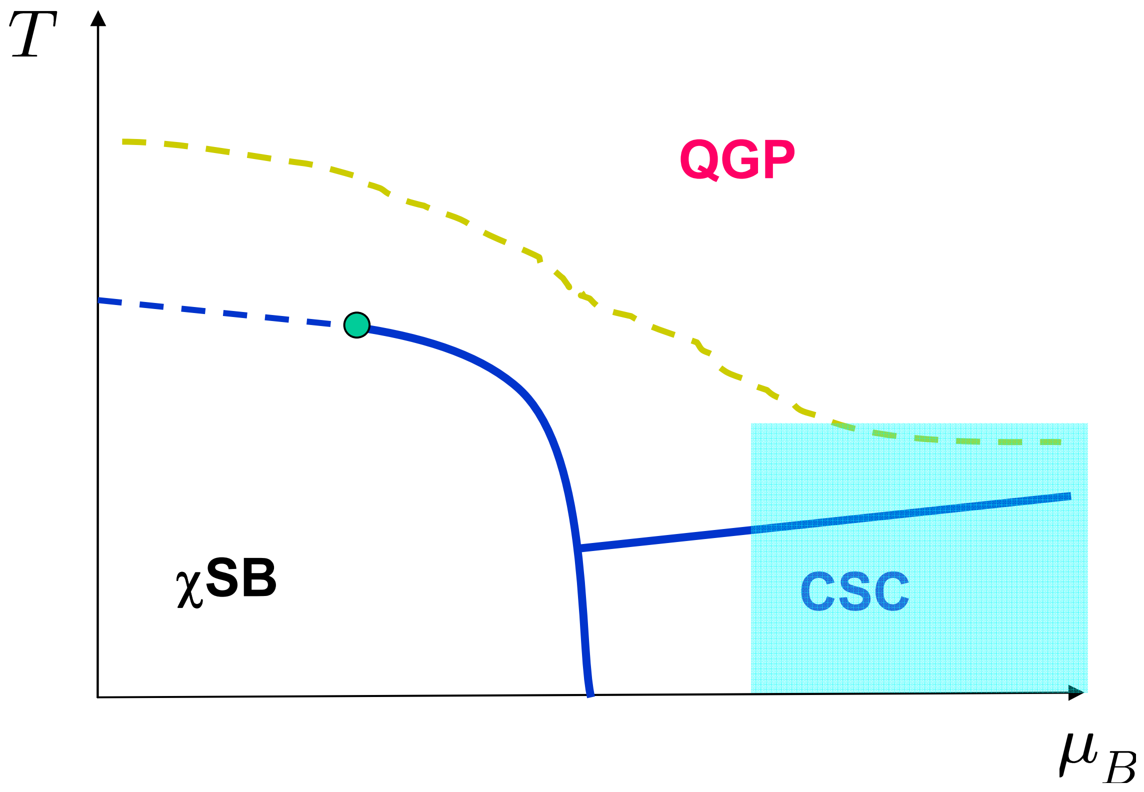
Heiselberg & Hjorth-Jensen, Phys.Rept.328 ('00)



(Coulomb + surface) energy

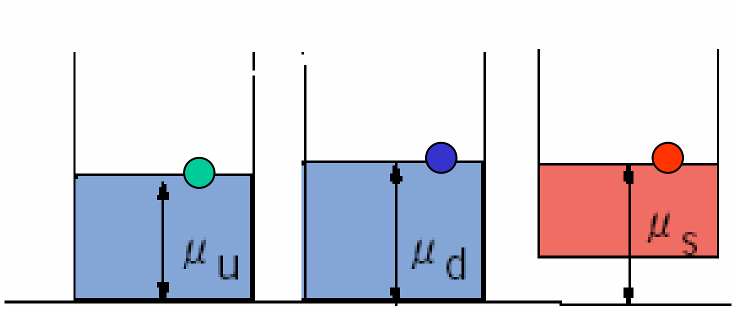
vs

symmetry energy



Bailin & Love, Phys. Rep. 107 (1984)
Iwasaki & Iwado, PL B350 (1995)
Alford, Rajagopal & Wilczek, PL B422 (1998)
Rapp, Schafer, Shuryak & Velkovsky, PRL81 (1998)

Origin of Color Superconductivity (CSC)



$$\Delta_{i\alpha} = \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\langle q_{j,\beta} C\gamma_5 q_{k,\gamma}\rangle$$

flavor color

Major differences from BCS

1. Highly **relativistic**
   Long range **magnetic int.**

$$|\Delta| \sim \varepsilon_F \; e^{-c/\sqrt{\alpha_s}}$$

High Tc superconductor

T_c/ε_F ~ 0.1

Compact Cooper pair

size ~ 1-10 fm

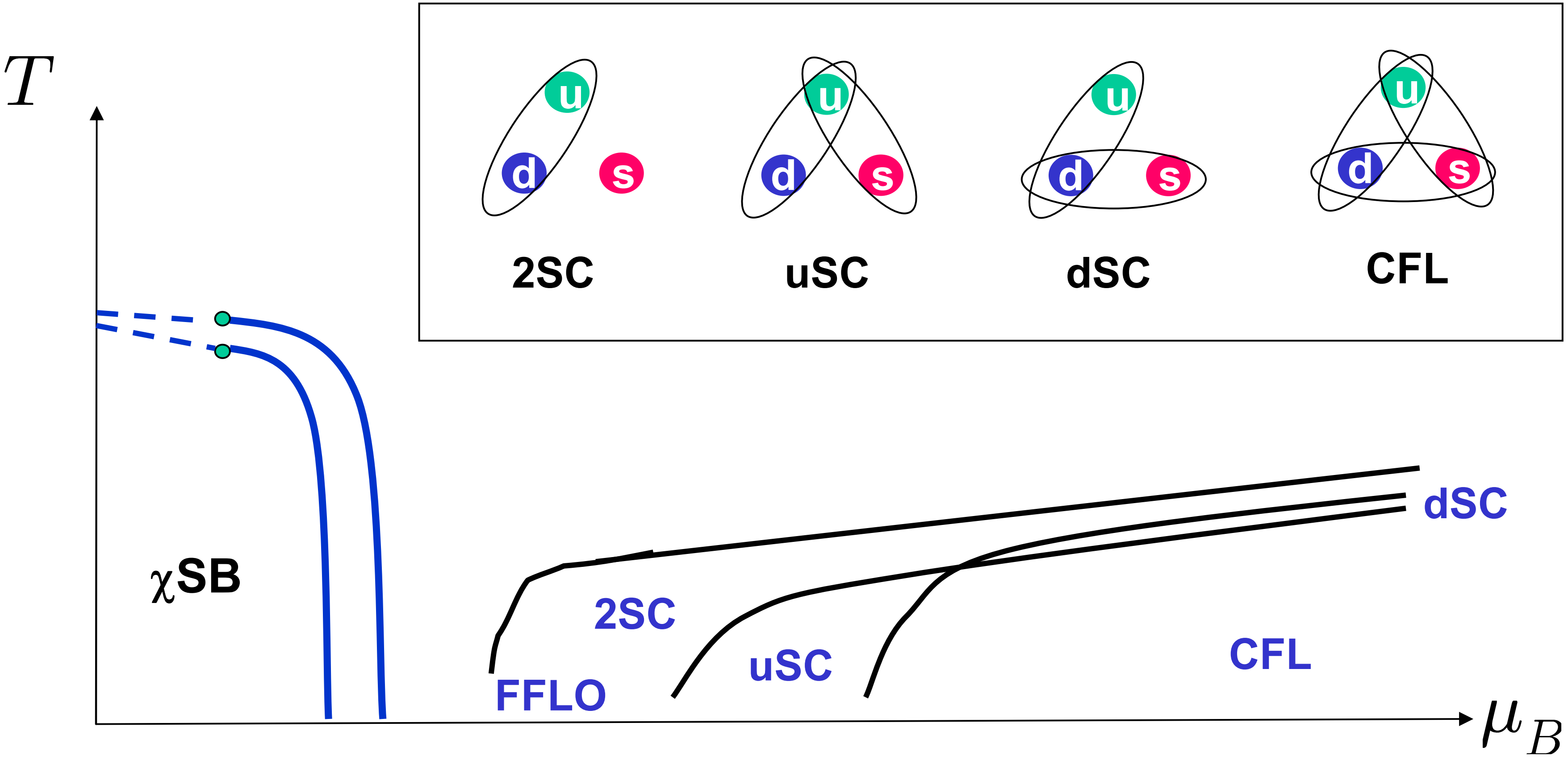
2. Color-flavor **entanglement**

$$\Delta_{i\alpha} = \begin{pmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{pmatrix}$$

Variety of phases
(such as ice and ^{3}He)
CFL, 2SC, dSC, uSC, etc

Phase structure relevant to compact stars

with charge-neutrality & β-equilibrium $\mathbf{n_d > n_u > n_s}$



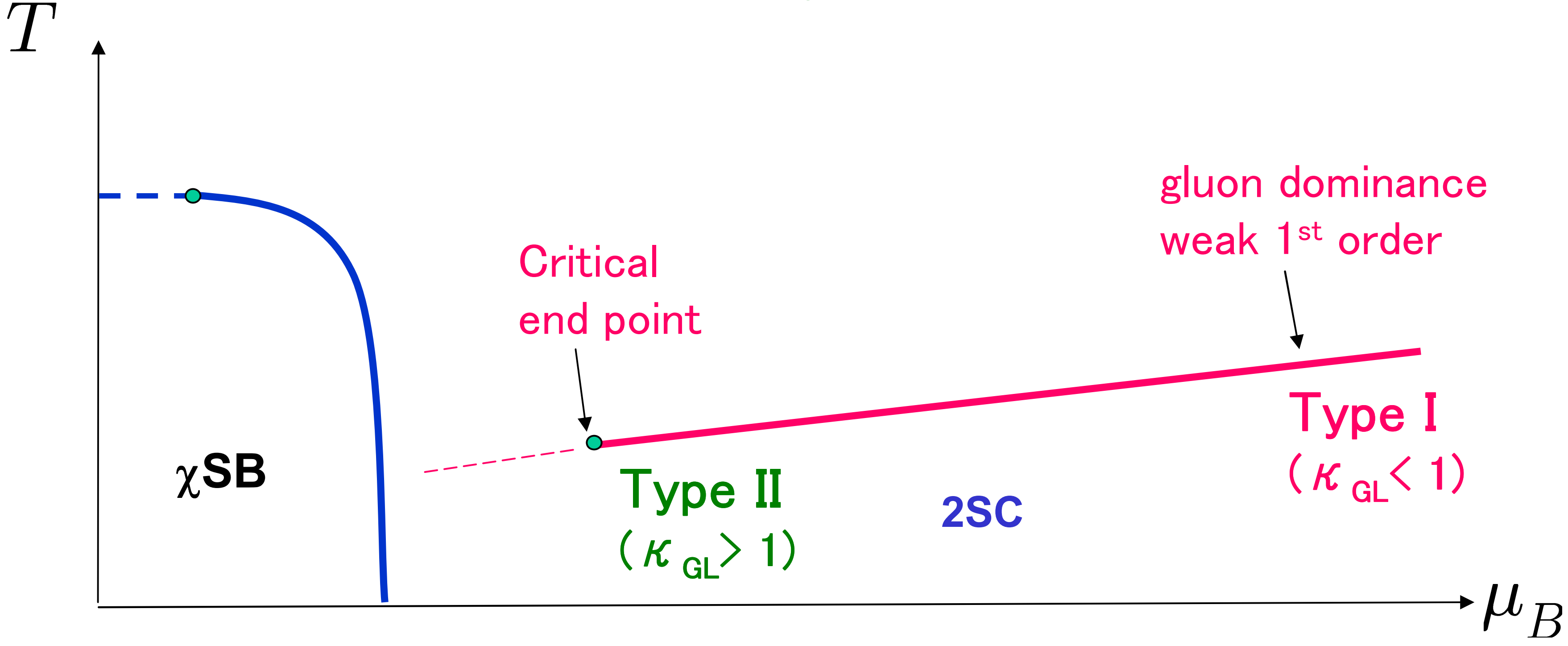
***2SC* :** Bailin and Love, Phys. Rep. ('84)
***CFL* :** Alford, Rajagopal and Wilczek, NPB ('99)
***dSC* :** Iida, Matsuura, Tachibana and Hatsuda, PRL ('04)
***uSC* :** Ruster, Werth, Buballa, Shovkovy and Rischke, PRD ('05)
FFLO, gapless phase, CSL, K-cond. etc

Thermal transition : 2SC → normal

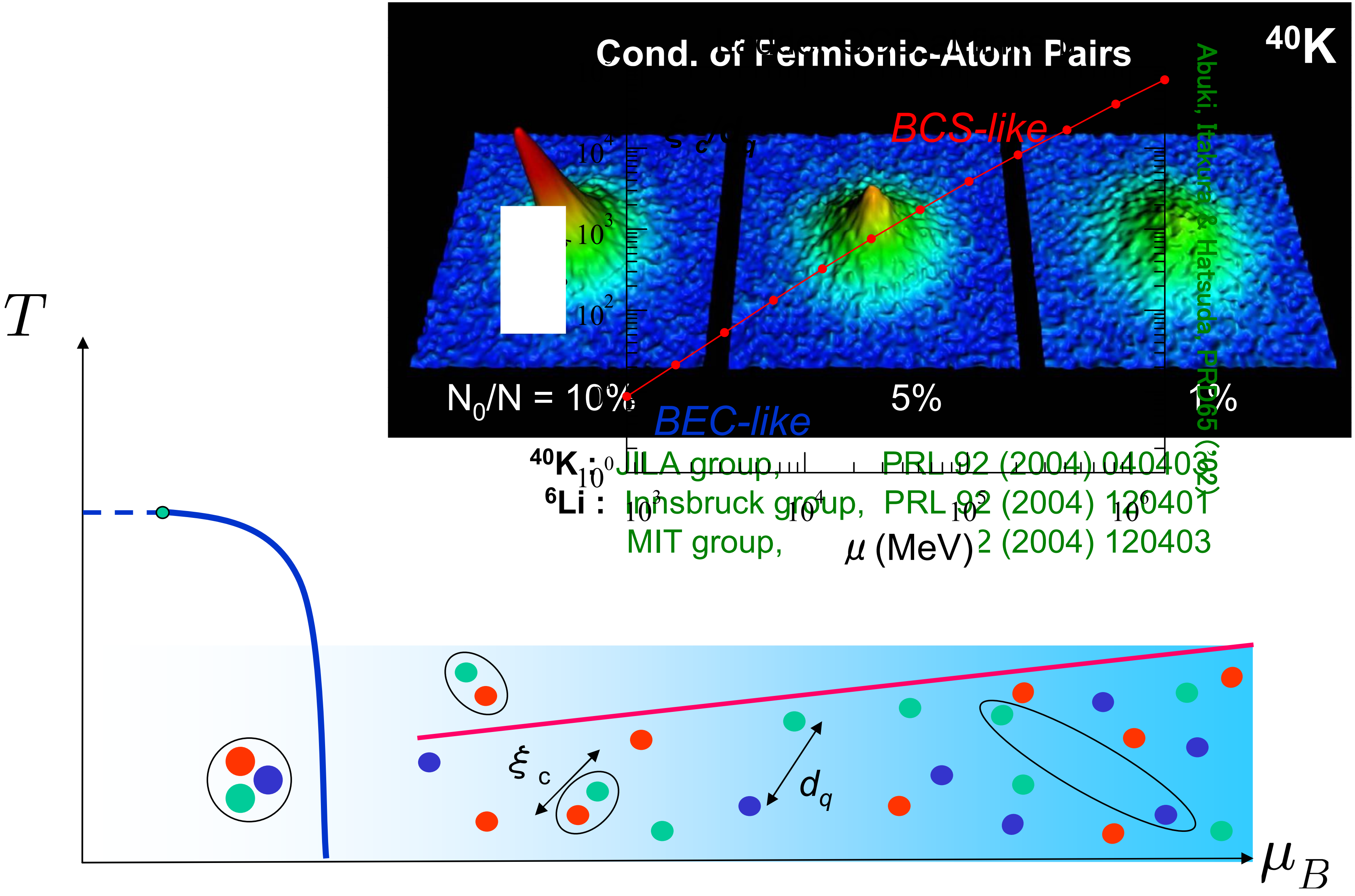
Ginzburg-Landau parameter :

$$\kappa_{\mathrm{GL}} = \frac{\text{color penetration depth}}{\text{color coherence length}} = \frac{m_\Delta}{m_A} \sim \frac{1}{\sqrt{\alpha_s}} \left(\frac{T_c}{\mu_B} \right)$$

- Weak coupling analyses Matsuura, Iida, Hatsuda & Baym, PRD 69 ('04)
- Lattice simulations : Digal, Hatsuda & Ohtani, hep-lat/0511018



BCS-BEC crossover ?



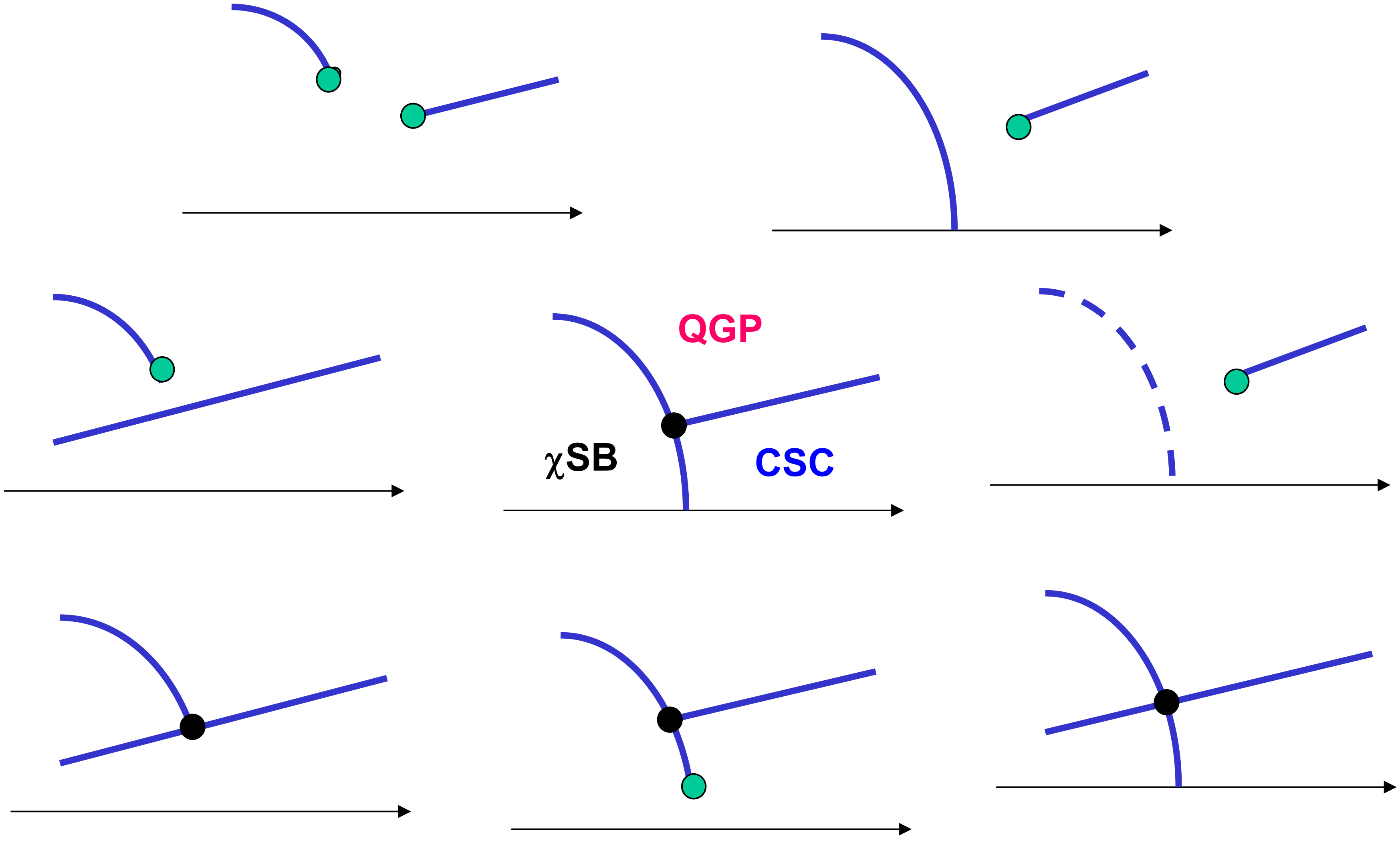
Abuki, Itakura & Hatsuda, PRD65 ('02)

^{40}K : JILA group, PRL 92 (2004) 040403

^{6}Li : Innsbruck group, PRL 92 (2004) 120401

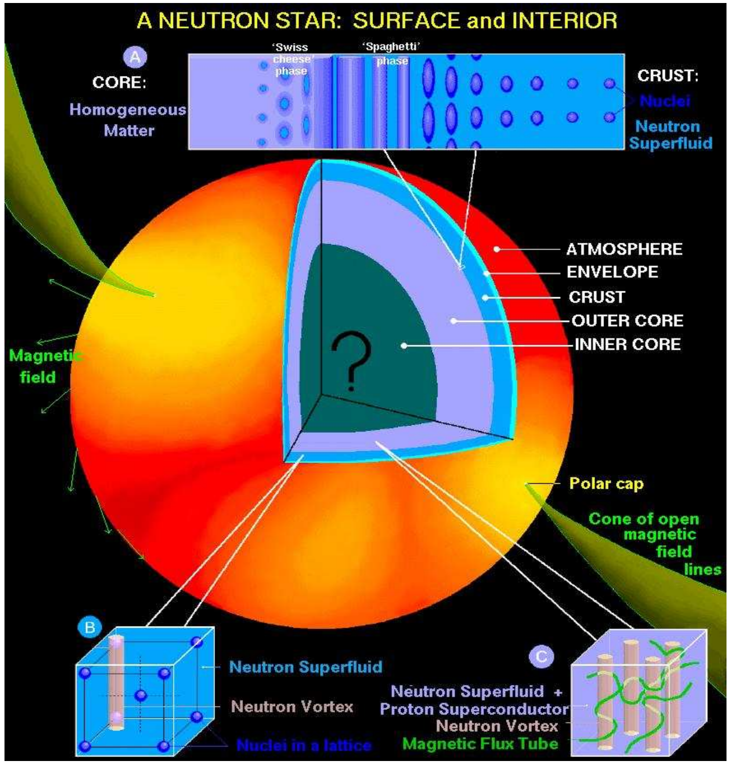
MIT group, PRL 92 (2004) 120403

Possible phase structure



See Kitazwa, Koide, Kunihiro & Nemoto, PTP108 ('02)

Probing Dense QCD

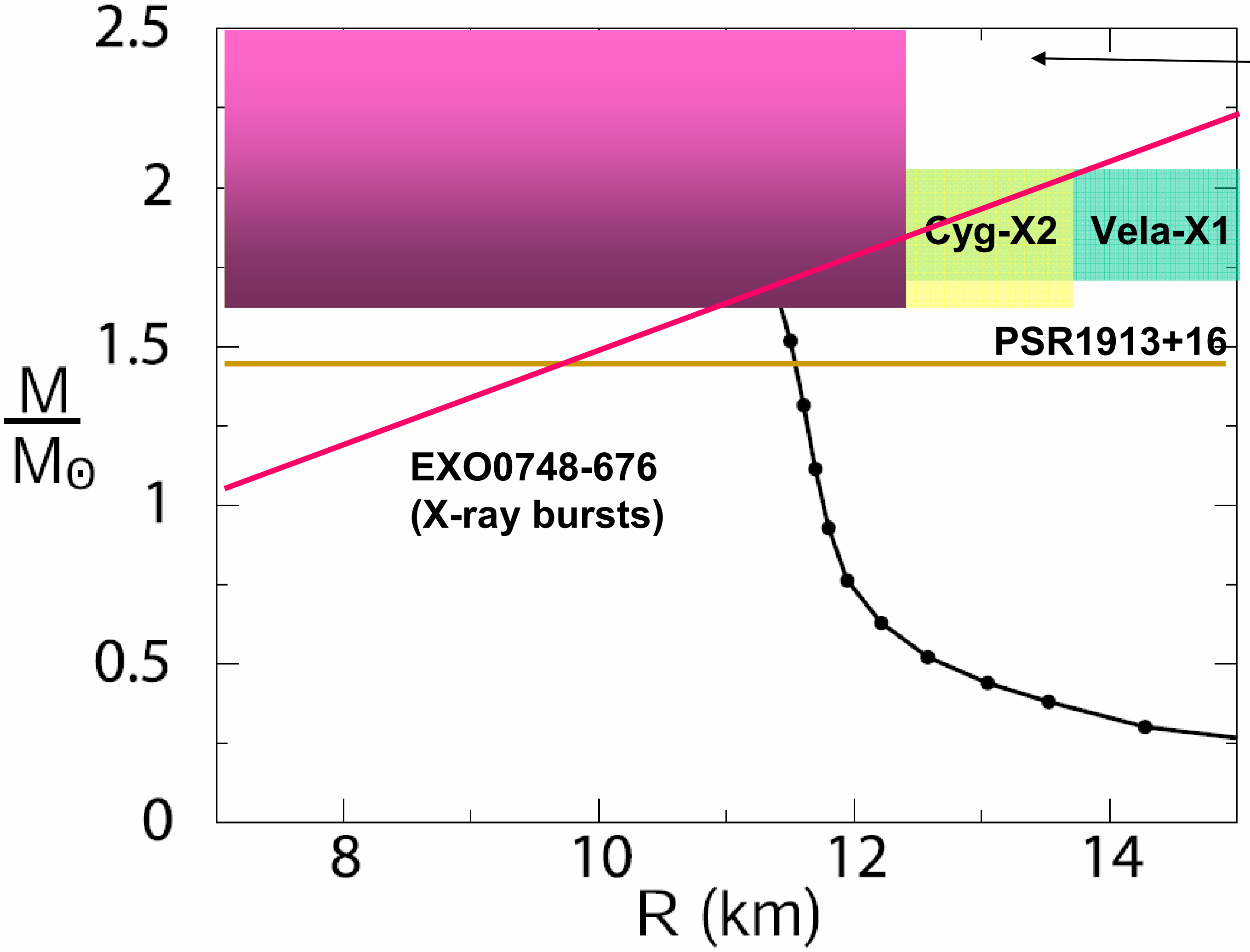


J-PARC

SIS100/300

M-R relation in APR EoS

$(\rho_{max} \sim 6\rho_0)$

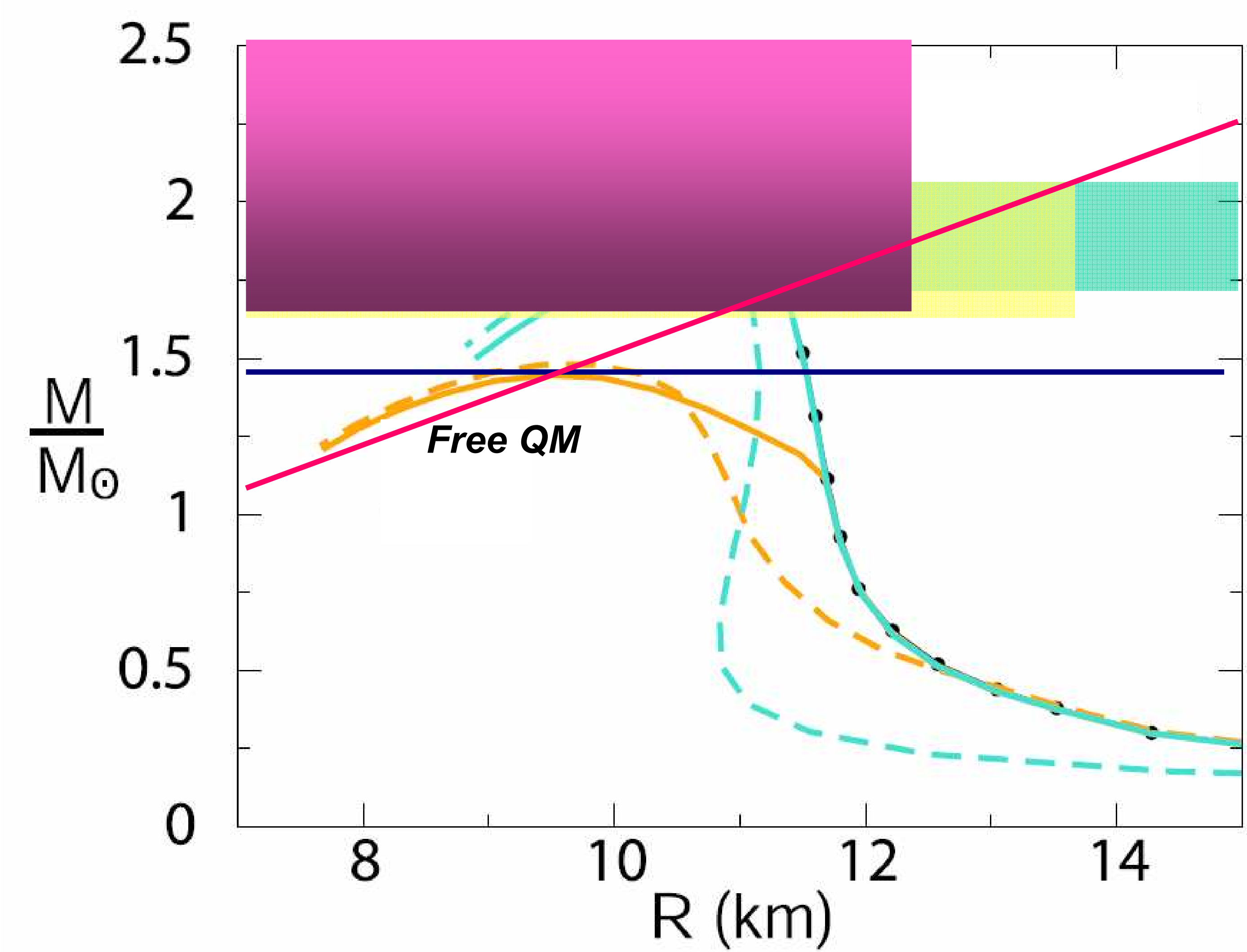


Neutron star - WD binary

X-ray binaries

Neutron star binary

M-R relation in APR EoS + CFL quark matter

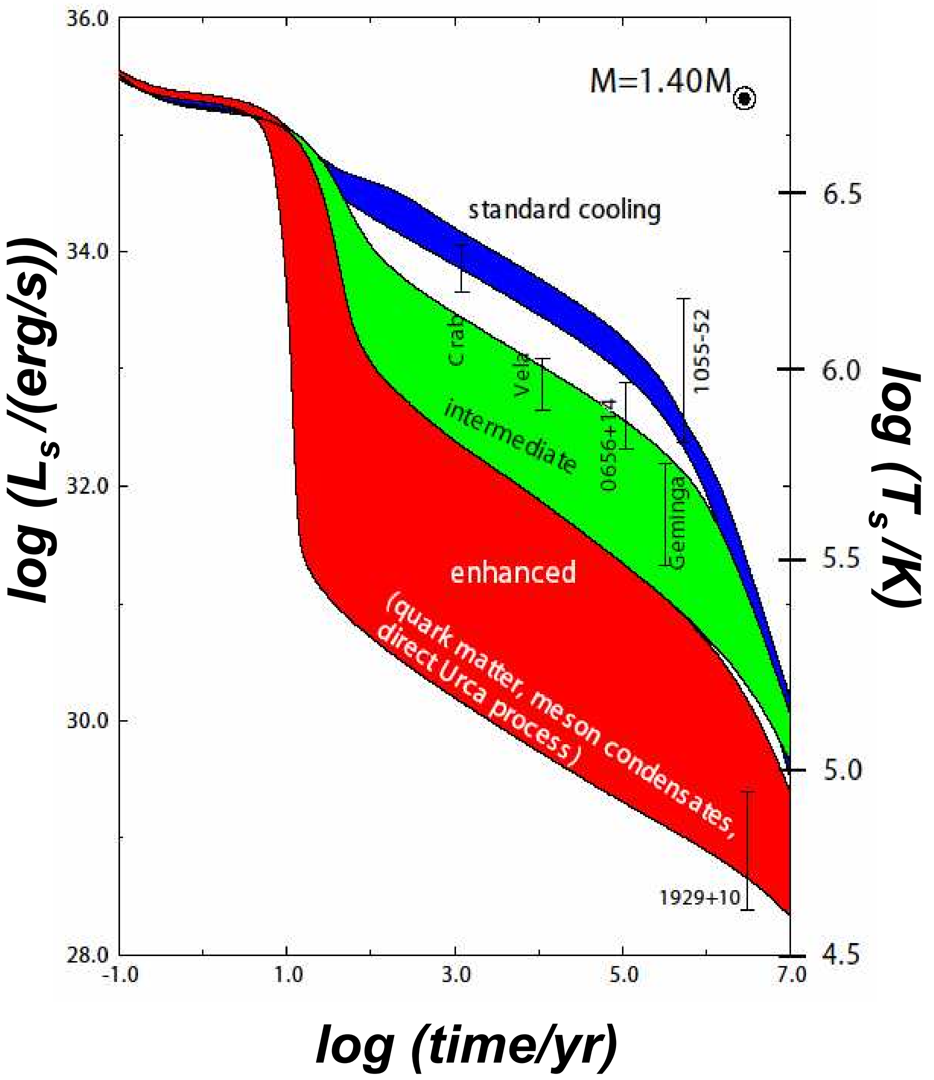


Alford, Braby, Paris & Reddy, ApJ 629 (2005)

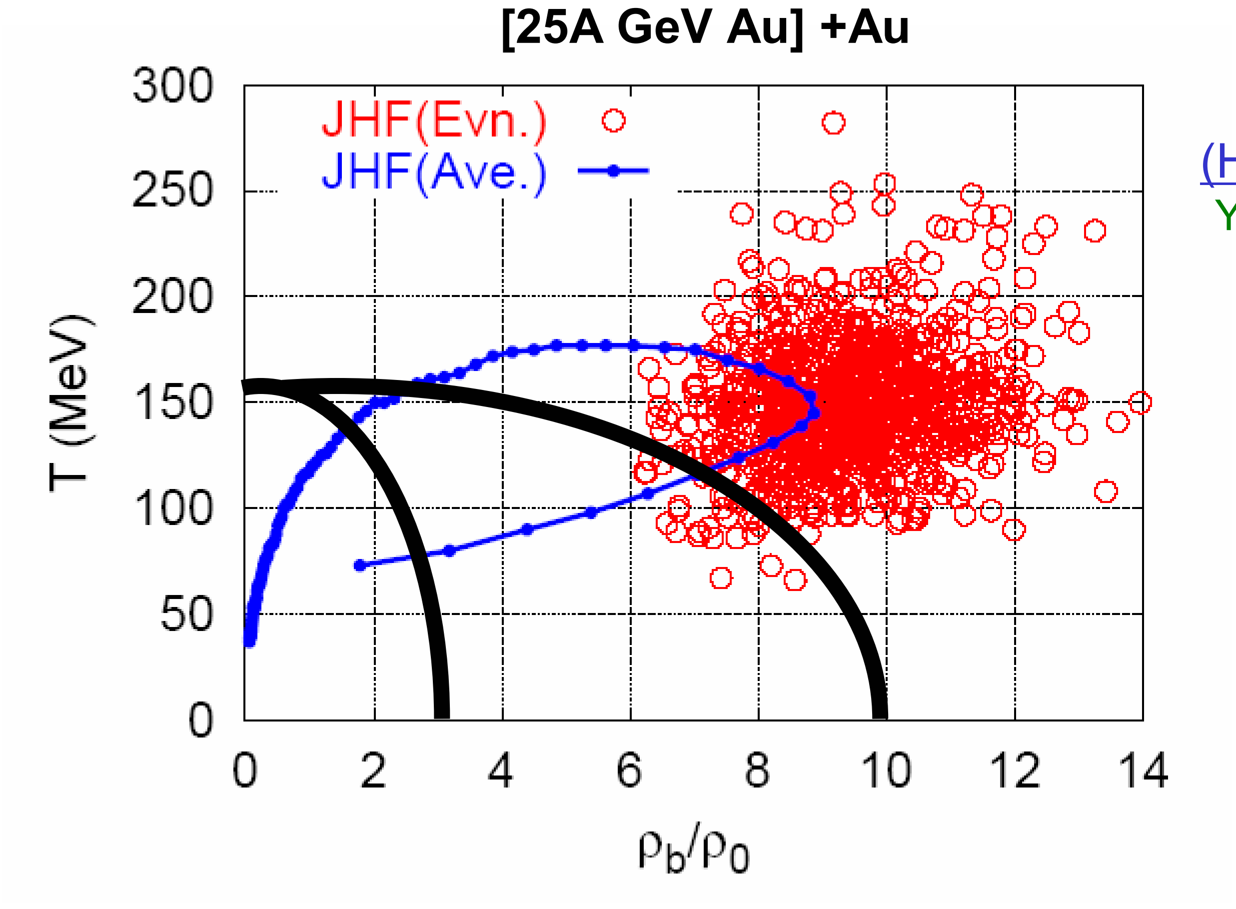
Cooling of neutron stars

	Name
Standard	Modified Urca
	Direct Urca
Exotic	Quark modified Urca
	Quark direct Urca
	π^- condensate
	K^- condensate
quenching	***n superfluidity***
	Q color super

Weber, astr-ph/0407155 ('04)



Heavy ion collision at J-PARC & SIS energies



JAM
(Hadronic cascade model)
Y. Nara et al, PRC61 (2000)

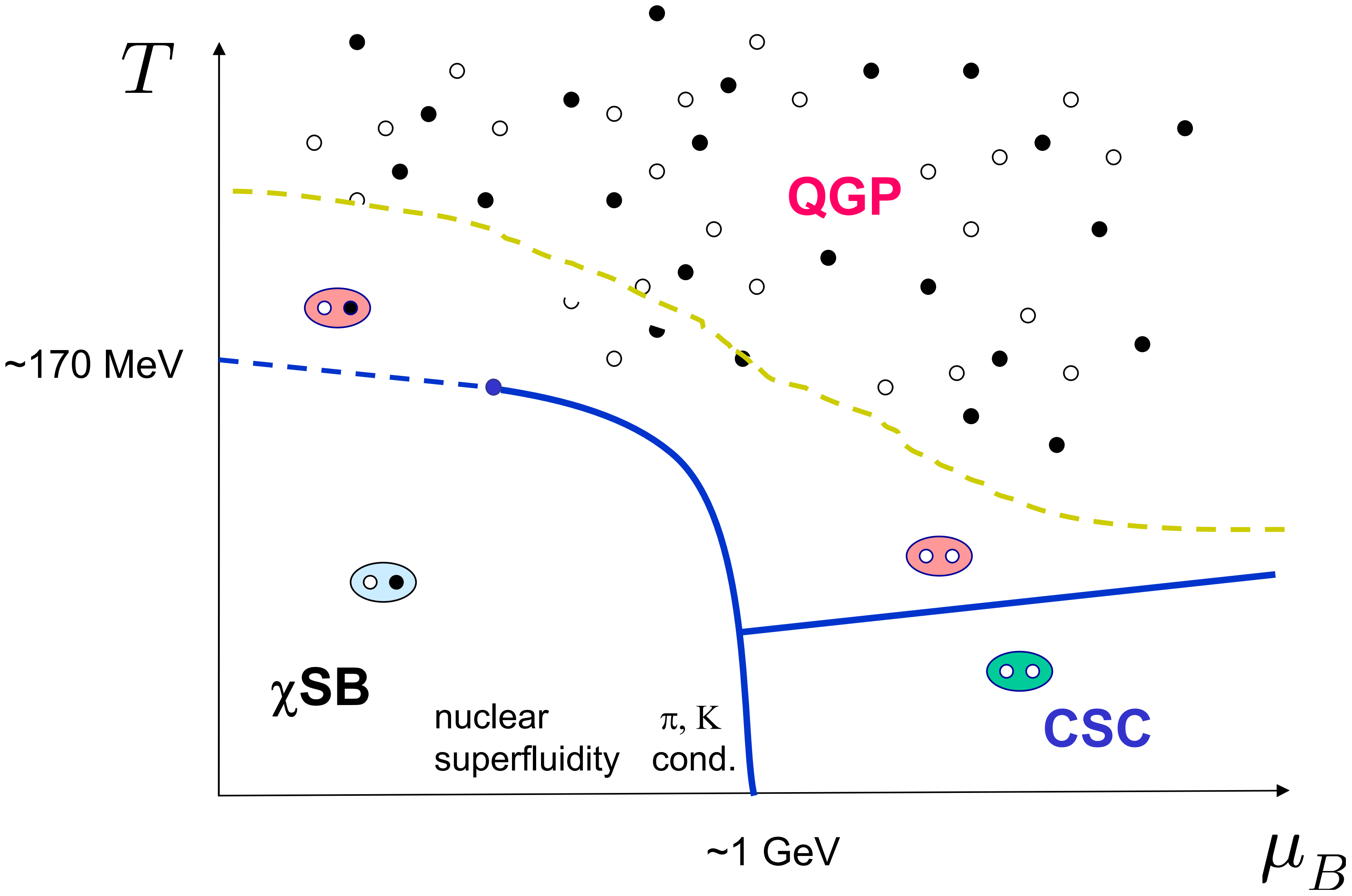
$\rho_B > 6\ \rho_0$

for about 3 fm/c

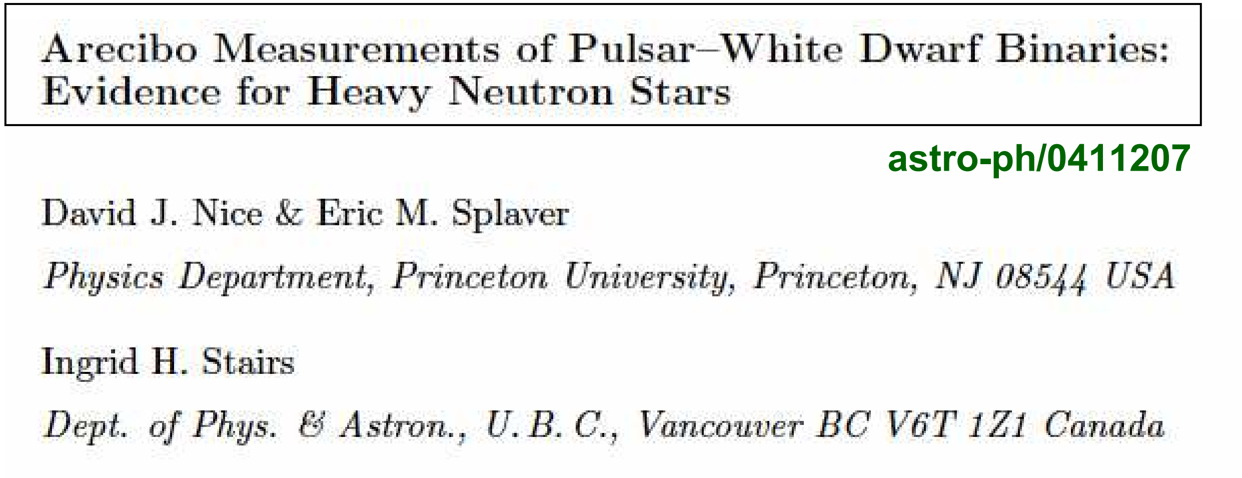
Figure taken from JHF report (2002) by A.Ohnishi

Summary

"phase"	theory	exp./obs.
χSB (low T & low μ)	Mature (precision physics) lattice QCD effective theories	variety of data
QGP (high T)	Developing lattice QCD effective theories	data accumulation RHIC → LHC
QM and CSC (low T & high μ)	Developing effective theories Need lattice inputs	• Neutron stars (M-R) • SIS, J-PARC, Nuclotron
"PG" (intermediate T & μ)	Not- fully explored Interesting connection to HTS, cold atoms	Relevant region to RHIC, SIS, J-PARC, Nuclotron ?


T
~170 MeV
QGP
χSB
nuclear
superfluidity
π, K
cond.
CSC
~1 GeV
μ_B

Back up slides



Arecibo Measurements of Pulsar–White Dwarf Binaries: Evidence for Heavy Neutron Stars

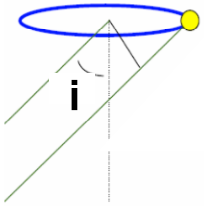
astro-ph/0411207

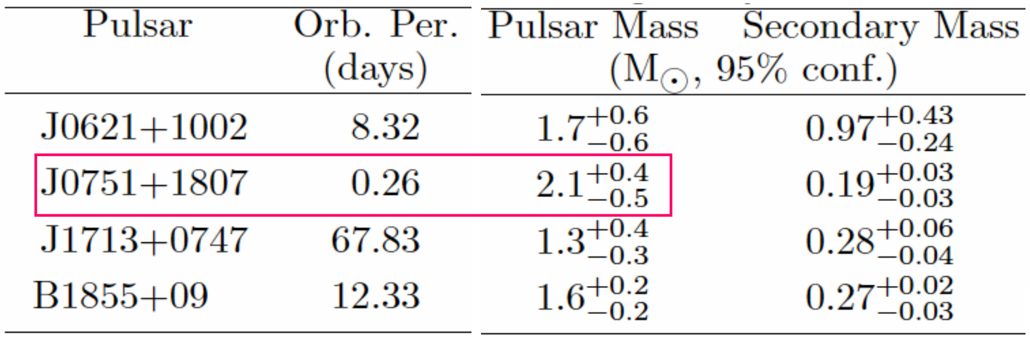
David J. Nice & Eric M. Splaver

Physics Department, Princeton University, Princeton, NJ 08544 USA

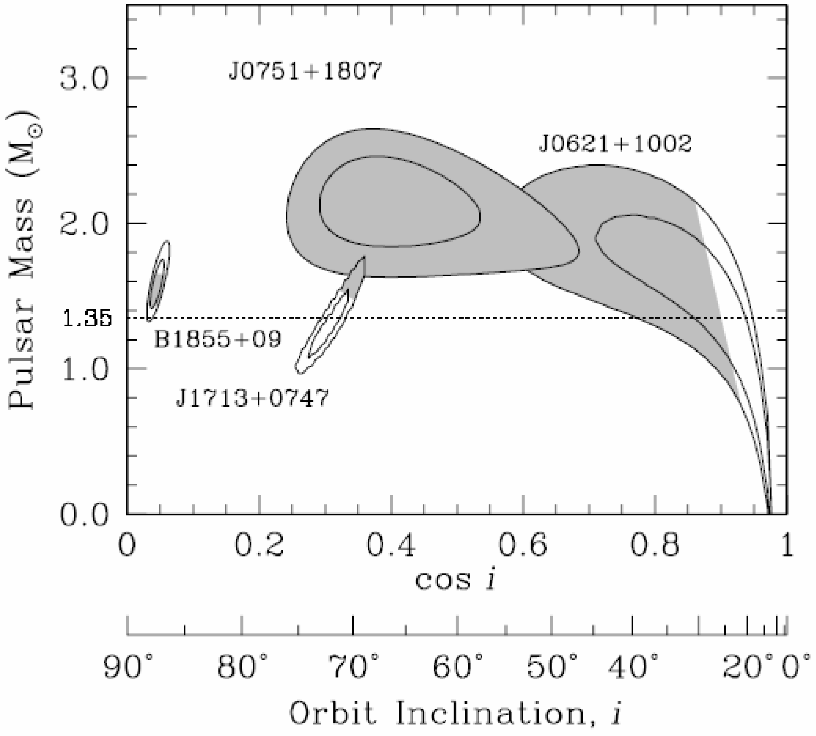
Ingrid H. Stairs

Dept. of Phys. & Astron., U.B.C., Vancouver BC V6T 1Z1 Canada





Pulsar	Orb. Per. (days)	Pulsar Mass	Secondary Mass
		(M$_\odot$, 95% conf.)	
J0621+1002	8.32	$1.7^{+0.6}_{-0.6}$	$0.97^{+0.43}_{-0.24}$
J0751+1807	0.26	$2.1^{+0.4}_{-0.5}$	$0.19^{+0.03}_{-0.03}$
J1713+0747	67.83	$1.3^{+0.4}_{-0.3}$	$0.28^{+0.06}_{-0.04}$
B1855+09	12.33	$1.6^{+0.2}_{-0.2}$	$0.27^{+0.02}_{-0.03}$



Gravitationally redshifted absorption lines in the X-ray burst spectra of a neutron star

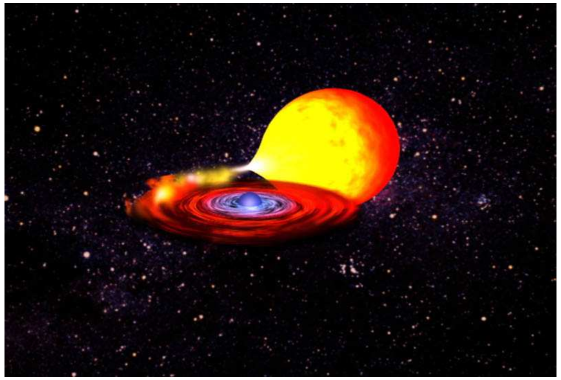
Nature 420 ('02)

J. Cottam*, F. Paerels†, & M. Mendez‡

*NASA Goddard Space Flight Center, Laboratory for High Energy Astrophysics, Greenbelt, MD, 20771, USA

†Columbia Astrophysics Laboratory and Department of Astronomy, Columbia University, 538 W. 120th St., New York, NY 10027, USA

‡SRON National Institute for Space Research, Sorbonnel Netherlands

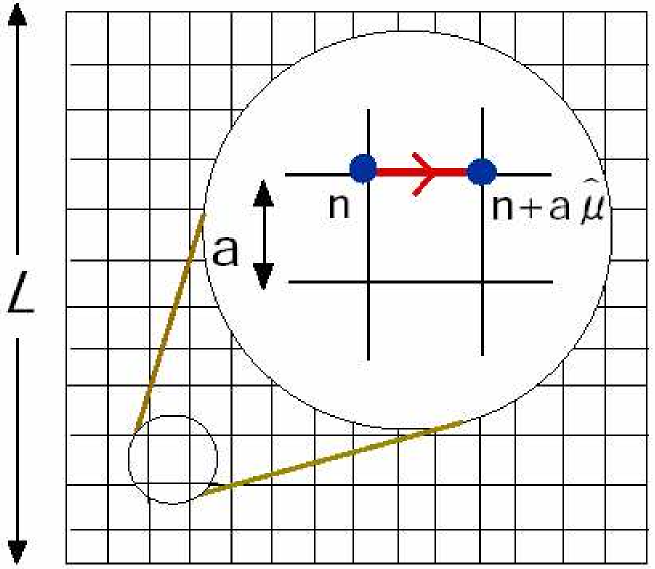


The fundamental properties of neutron stars provide a direct test of the equation of state of cold nuclear matter, a relationship between pressure and density that is determined by the physics of the strong interactions between the particles that constitute the star. The most straightforward method of determining these properties is by measuring the gravitational redshift of spectral lines produced in the neutron star photosphere[1]. The equation of state implies a mass-radius relation, while a measurement of the gravitational redshift at the surface of a neutron star provides a direct constraint on the mass-to-radius ratio. Here we report the discovery of significant absorption lines in the spectra of 28 bursts of the low-mass X-ray binary EXO 0748−676. We identify the most significant features with the Fe XXVI and XXV $n = 2$–3 and O VIII $n = 1$–2 transitions, all with a redshift of $z = 0.35$, identical within small uncertainties for the respective transitions. For an astrophysically plausible range of masses ($M \sim 1.3$–$2.0\ M_{\odot}$)[2-5], this value is completely consistent with models of neutron stars composed of normal nuclear matter, while it excludes some models[6-7] in which the neutron stars are made of more exotic matter.

$$\frac{M}{M_{\odot}} = 1.5\ \frac{R}{10\mathrm{km}}$$

QCD on the Lattice

4-d Euclidean lattice



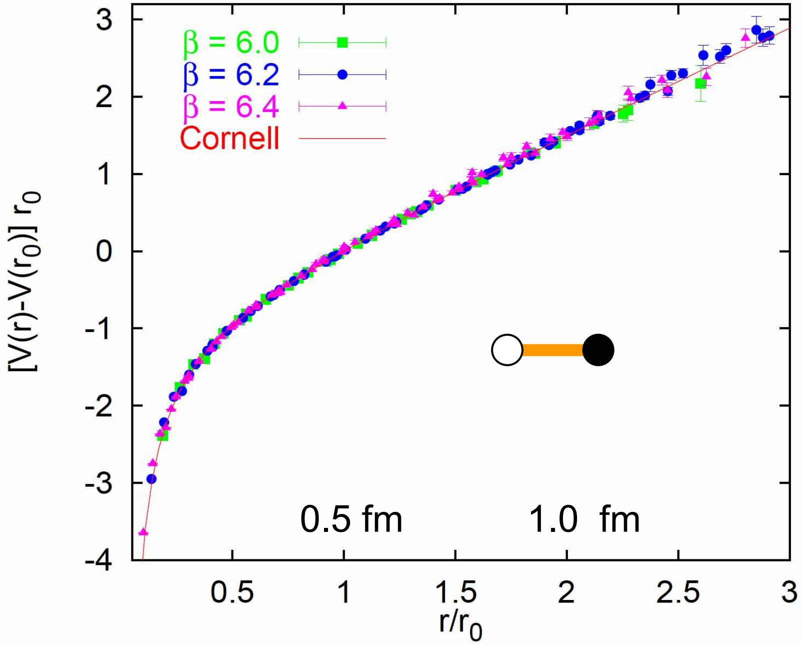
Discretization

$A_\mu(x) \rightarrow U_\mu(n) = \exp(i a A_\mu)$

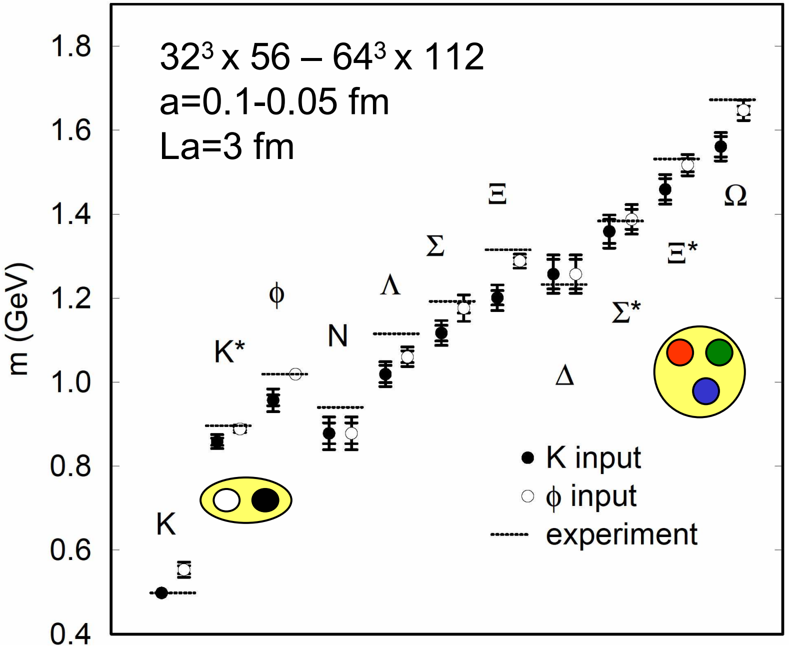
$q(x) \rightarrow q(n)$

$T=0 : L^3 \times L = (N_s a)^4$

$T\neq0 : L^3 \times 1/T = (N_s a)^3 \times (N_t a)$

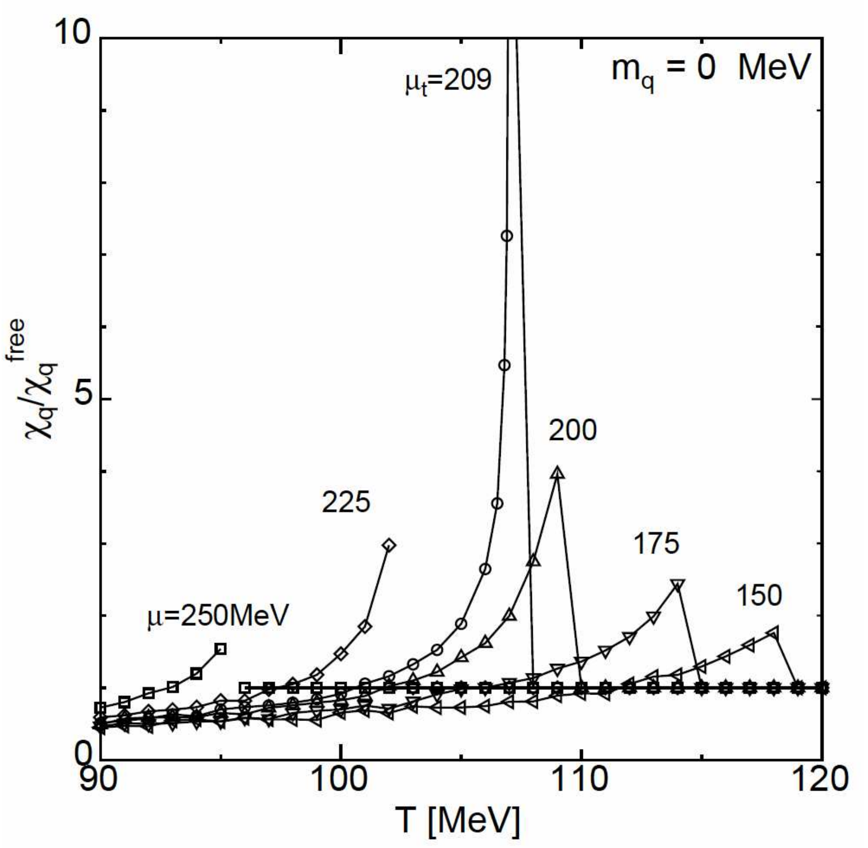


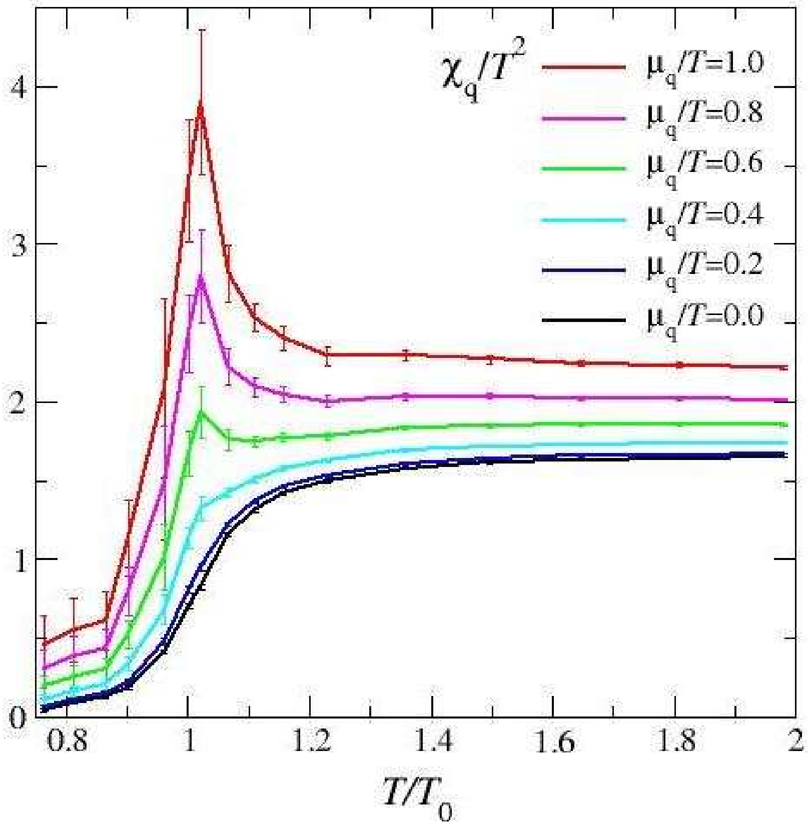
Bali, Phys. Rep. 343 ('01) 1



CP-PACS, PRD 67 ('03) 034503

Quark number susceptibility





$$\frac{\chi_{\rm q}}{T^4}\left(\mu_{\rm q}\right)=+2c_2+12c_4\left(\frac{\mu_{\rm q}}{T}\right)^2+O\left(\mu_{\rm q}^4\right)$$

Kunihiro, Phys. Lett. B271 ('92) 395
Hatta-Ikeda, Phys.Rev.D67 ('03) 014028

Ejiri et al. (Bielefeld-Swansea Coll.)

Lattice QCD simulations

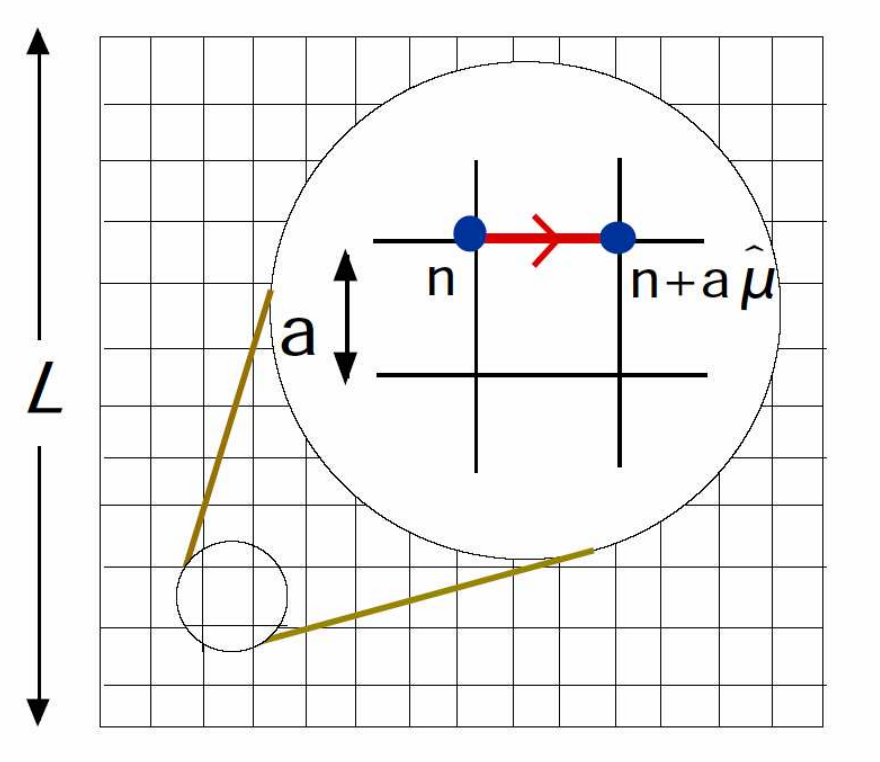
Discretization

$A_\mu(x) \rightarrow U_\mu(n) = \exp(i\, a\, A_\mu)$

$q(x) \rightarrow q(n)$

$T=0 : L^3 \times L$ hypercube

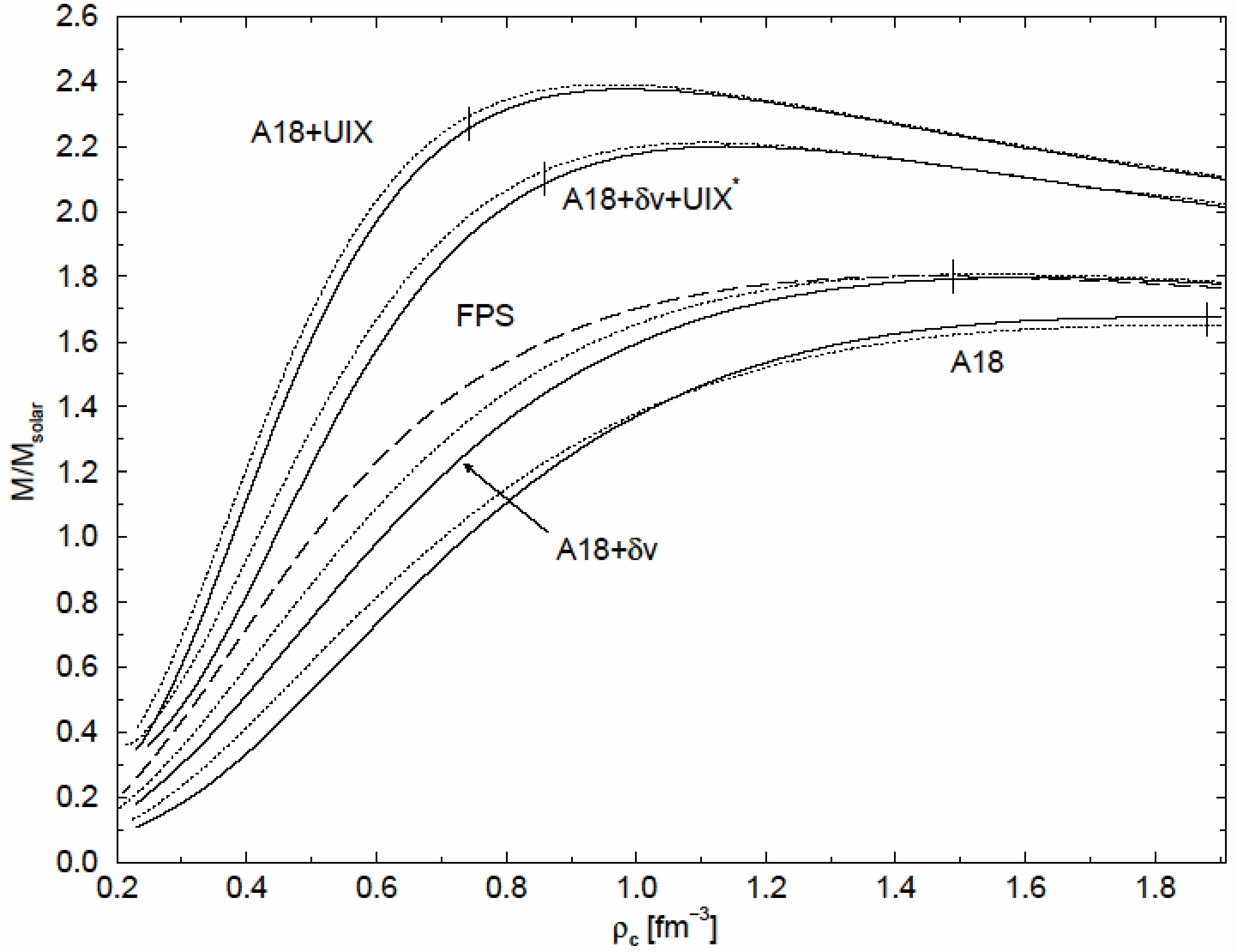
$T\neq 0 : L^3 \times 1/T$ slab



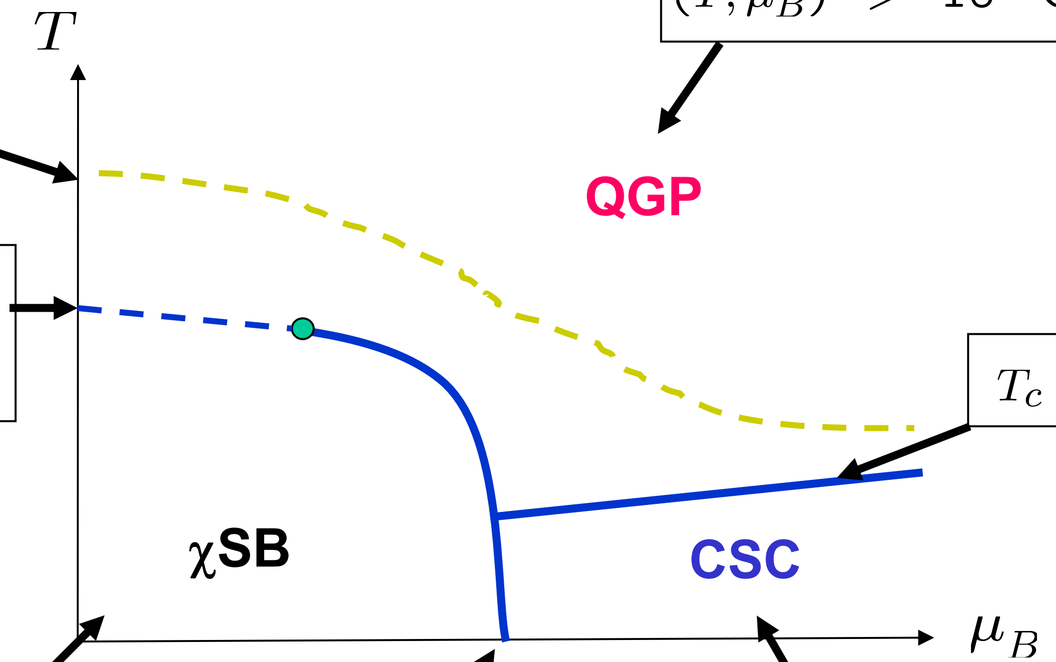
$$Z = \mathrm{Tr}\left[e^{-(H-\mu N)/T}\right] = \int [dA]\ \mathrm{Det}[\hat{D} + m + i\mu\gamma_4]\ e^{-S(A)}$$

Complex

Sign problem !


A18+UIX
A18+δv+UIX*
FPS
A18
A18+δv
M/M_solar
ρ_c [fm^-3]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

Scale of each “phase”



$g < 1$
$(T, \mu_B) > 10^3$ GeV

$T^* \sim 2T_c$?

$T_c \sim (\sqrt{3}/\pi)\ M$
~ 170 MeV

$T_c \sim 0.57\Delta_{T=0}$

$E = \sqrt{\mathbf{p}^2 + M^2}$
$M \sim 300\text{MeV} \sim \Lambda_{\text{QCD}}$

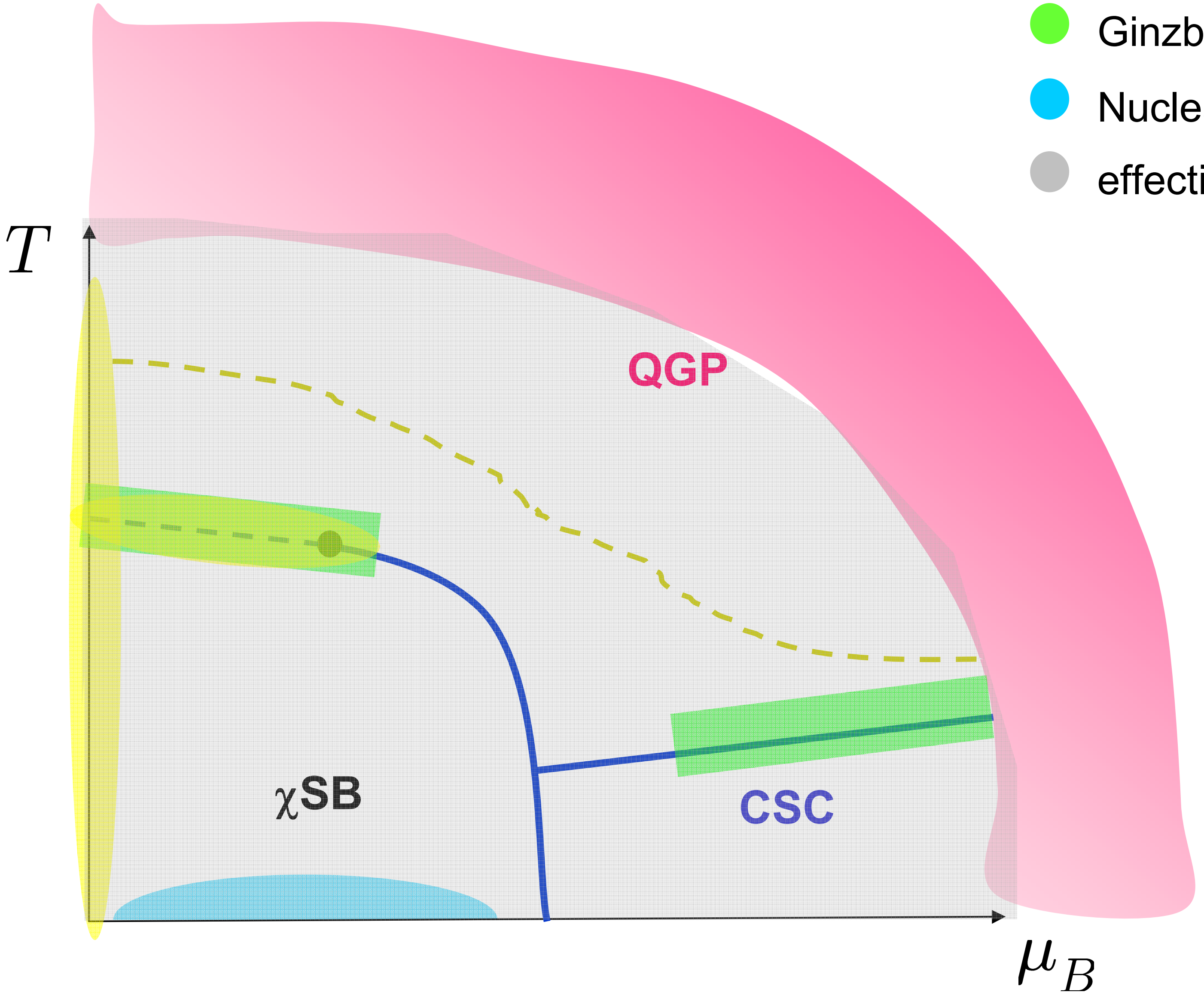
$\delta\mu_c = \mu_c - M_{\text{N}}$
$\sim \Lambda_{\text{QCD}}$?

$\rho_c = (3-10)\ \rho_0$?

$E = \sqrt{(p - p_{\text{F}})^2 + \Delta^2}$
$\Delta \sim 50\text{MeV}$?

Theoretical status

- ● Lattice QCD
- ● Perturbative QCD
- ● Ginzburg-Landau + RG
- ● Nuclear theory
- ● effective models



Order of the thermal transition (μ=0)

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

O(4) GL theory

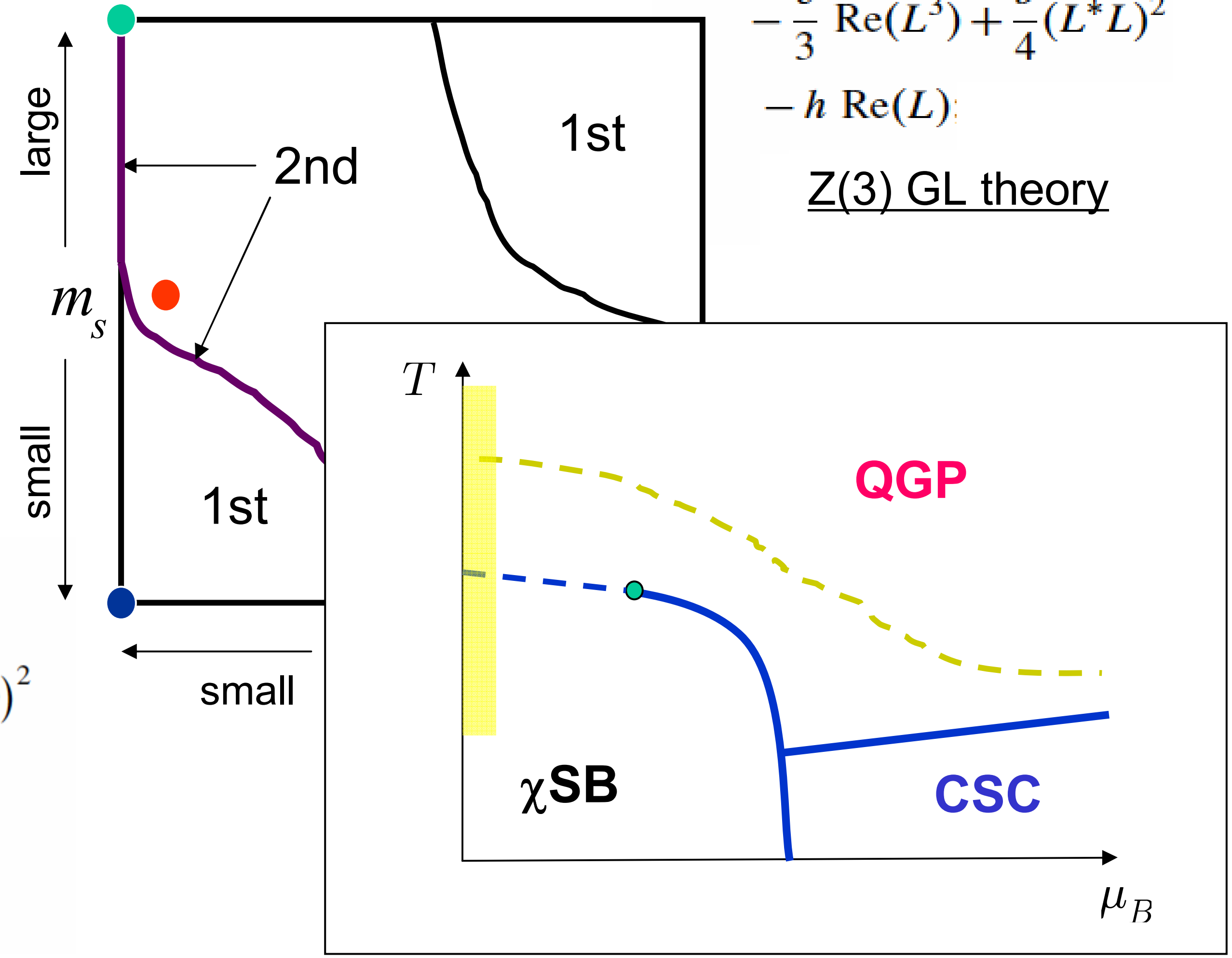
$$L_{GL} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{a(m_s,T)}{2}\vec{\phi}^2 + \frac{b(m_s,T)}{4!}(\vec{\phi}^2)^2 + \frac{c}{6!}(\vec{\phi}^2)^3 - h\phi_0$$

$$L_{GL} \quad \frac{1}{2}(\nabla L^*)(\nabla L) + \frac{a}{2}L^*L - \frac{c}{3}\,\mathrm{Re}(L^3) + \frac{b}{4}(L^*L)^2 - h\,\mathrm{Re}(L)$$

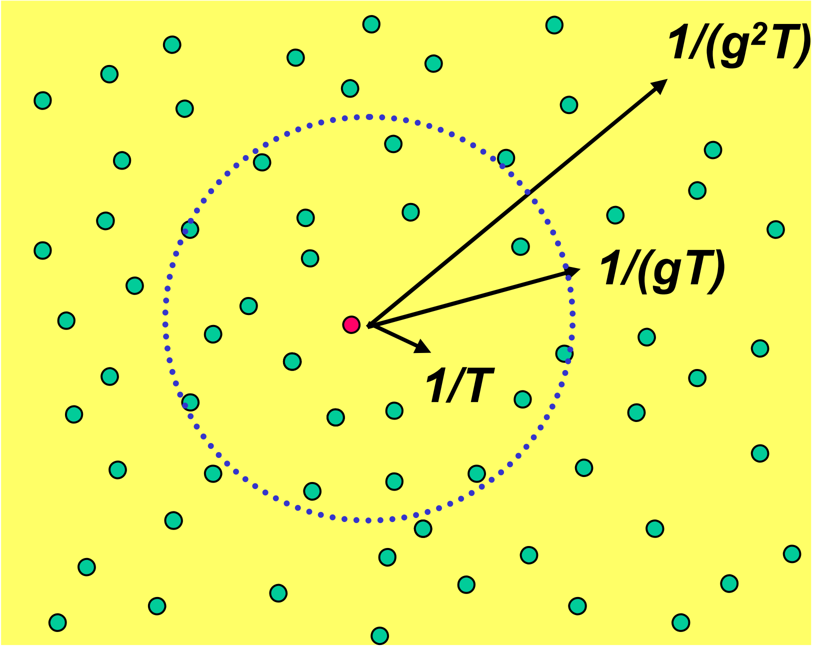
Z(3) GL theory

SU$_L$(3)xSU$_R$(3) GL theory

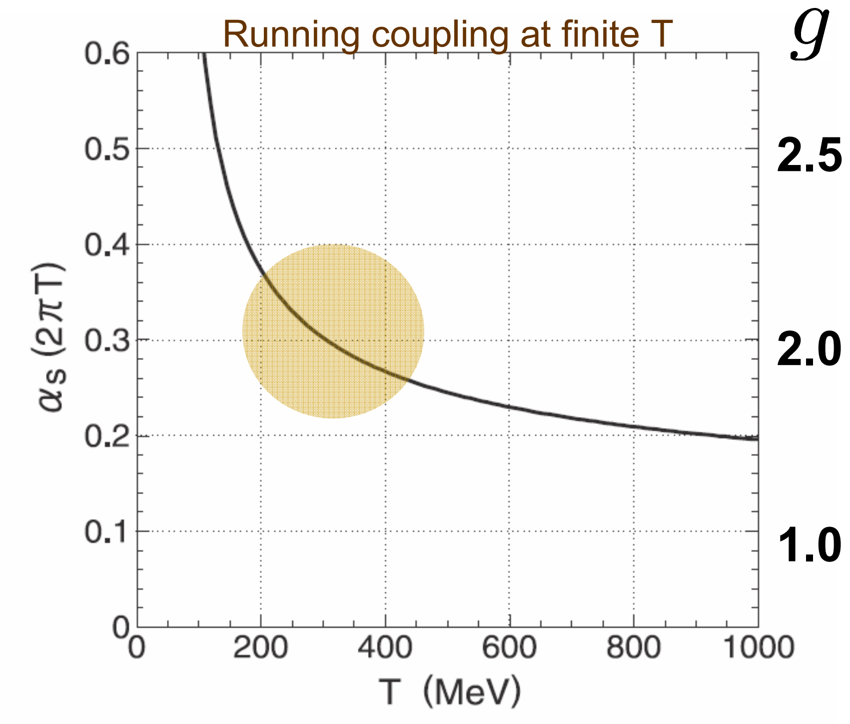
$$L_{GL} = \frac{1}{2}\,\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 - \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right) - \frac{1}{2}\,\mathrm{tr}\,h(\Phi+\Phi^\dagger).$$



Scale degeneracy near T_c

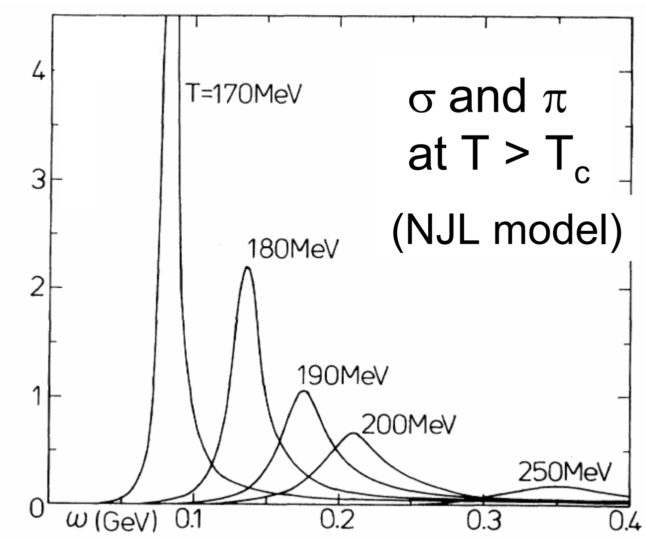


T^{-1}	Inter-particle distance
$(gT)^{-1}$	Electric screening length
$(g^2T)^{-1}$	Magnetic screening length

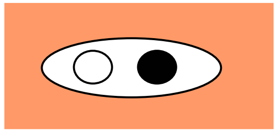


Pre-formed pairs (PFP) for $T_c < T < T^*$

Hatsuda & Kunihiro, PRL ('85)



σ and π
at $T > T_c$
(NJL model)

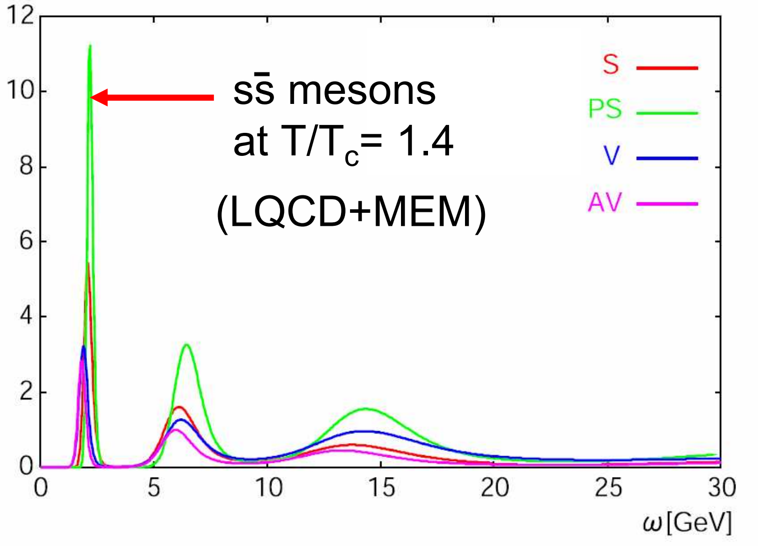


T_c	:	absence of coherence
T^*	:	dissociation of PFP

BEC-BCS crossover:

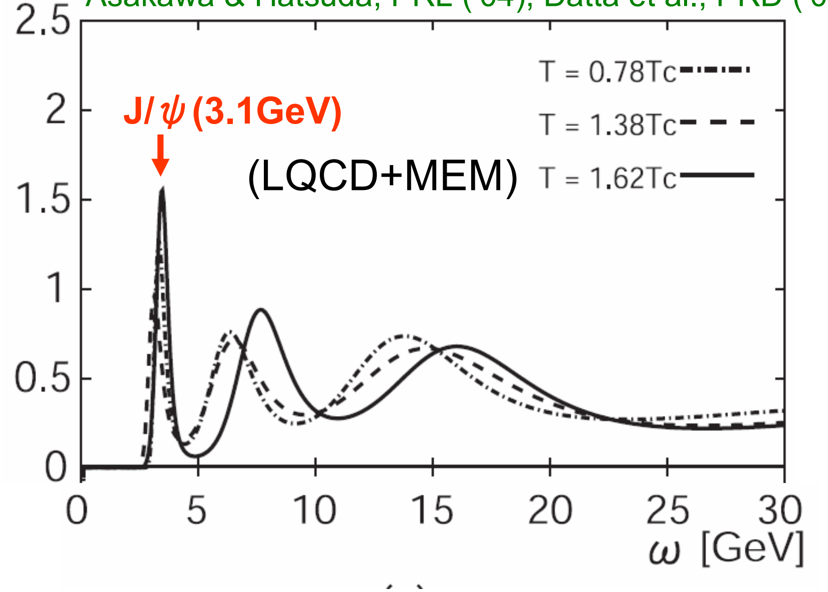
Leggett ('80), Nozieres & Schimitt-Rink ('85)

Asakawa & Hatsuda, PTP ('04)



$s\bar{s}$ mesons
at T/T_c= 1.4
(LQCD+MEM)

Asakawa & Hatsuda, PRL ('04), Datta et al., PRD ('04)

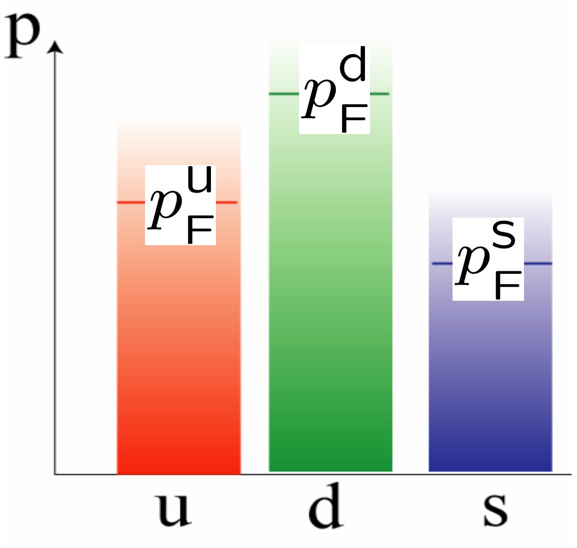


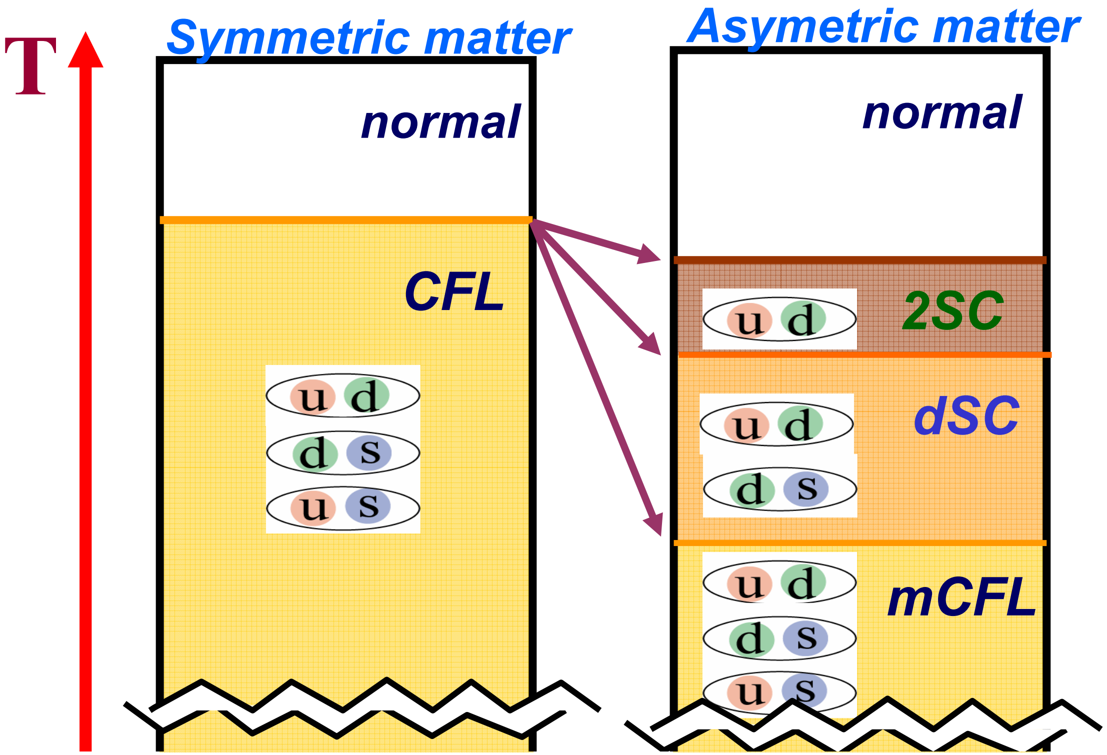
J/ψ (3.1GeV)

(LQCD+MEM)

Master formula near $T_{\rm c}$

$$\frac{T_{\rm c}^{ij}}{T_{\rm c}} = 1 + \frac{\frac{1}{2}(p_{\rm F}^i + p_{\rm F}^j) - \mu}{\mu} \ln\left(\frac{\mu}{T_{\rm c}}\right)$$





T

Symmetric matter

normal

CFL

Asymetric matter

normal

2SC

dSC

mCFL

Iida, Matsuura,
Tachibana and Hatsuda,
PRL ('04)