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ONE-ARMED SPIRAL INSTABILITY IN DIFFERENTIALLY ROTATING STARS

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ABSTRACT

We investigate the dynamical instability of the one-armed spiral $m = 1$ mode in differentially rotating stars by means of $3 + 1$ hydrodynamical simulations in Newtonian gravitation. We find that both a soft equation of state and a high degree of differential rotation in the equilibrium star are necessary to excite a dynamical $m = 1$ mode as the dominant instability at small values of the ratio of rotational kinetic to potential energy, $T/|W|$. We find that this spiral mode propagates outward from its point of origin near the maximum density at the center to the surface over several central orbital periods. An unstable $m = 1$ mode triggers a secondary $m = 2$ bar mode of smaller amplitude, and the bar mode can excite gravitational waves. As the spiral mode propagates to the surface it weakens, simultaneously damping the emitted gravitational wave signal. This behavior is in contrast to waves triggered by a dynamical $m = 2$ bar instability, which persist for many rotation periods and decay only after a radiation-reaction damping timescale.

Subject headings: Gravitation — hydrodynamics — instabilities — stars: neutron — stars: rotation

We have studied the conditions under which Newtonian, differentially rotating stars are dynamically unstable to an $m = 1$ one-armed spiral instability, and found that both soft equations of state and a high degree of differential rotation are necessary to trigger the instability. For sufficiently soft equations of state and sufficiently high degrees of differential rotation we found that stars are dynamically unstable even at the small values of the ratio of rotational to potential energy $T/|W| \sim 0.14$ considered in this paper.

While we find that a toroidal structure alone is not sufficient for the $m = 1$ instability, all the models that are unstable do have a toroidal structure, suggesting that this may be a necessary condition. The growing $m = 1$ mode redistributes the matter in the unstable star and destroys the toroidal structure after a few central rotation periods.

Quasi-periodic gravitational waves emitted by stars with $m = 1$ instabilities have smaller amplitudes than those emitted by stars unstable to the $m = 2$ bar mode. For $m = 1$ modes, the gravitational radiation is emitted not by the primary mode itself, but by the $m = 2$ secondary harmonic which is simultaneously excited, albeit at a lower amplitude. Unlike the case for bar-unstable stars, the gravitational wave signal does not persist of many periods, but instead is damped fairly rapidly in most of the cases we have examined.

We have plotted typical wave forms for stars unstable to $m = 2$ bar modes and for stars unstable to one-armed spiral $m = 1$ modes. Characteristic wave frequencies $f_{\rm GW}$ are seen to be the order of central angular velocity $\sim \Omega_{\rm c}$, and are considerably higher than the equatorial angular velocity at the surface $\Omega_{\rm eq} \sim (M/R^3)^{1/2}$ due to appreciable differential rotation. For supermassive stars ($M \gtrsim 10^5 M_\odot$) the amplitudes and frequencies of these waves fall well within the detectable range of LISA.

A more detail discussion is presented in Saijo, Baumgarte, & Shapiro (2003)[1].

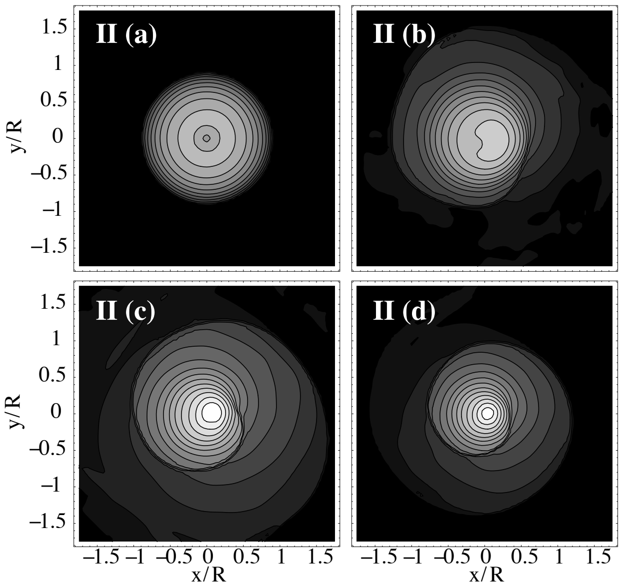

Fig. 1.— Final density contours in the equatorial plane for Models II in Saijo, Baumgarte, & Shapiro (2003)[1]. The stiffness of the equation of state decreases from model (a) to (d). Snapshots are plotted at $(t/P_{\rm c},\ \rho_{\rm max}/\rho^{(0)}_{\rm max},\ d)$ = (a) (36.8, 1.24, 0.220), (b) (23.8, 2.63, 0.267) (c) (17.3, 6.91, 0.333), and (d) (18.6, 9.09, 0.333). The contour lines denote densities $\rho/\rho_{\rm max} = 10^{-(16-i)d} (i = 1, \cdots, 15)$.

[1] Saijo, M., Baumgarte, T. W., & Shapiro, S. L. 2003, ApJ, submitted (astro-ph/0302436)